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The continuation of

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CONTENTS

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(1896-1948)

An International Quarterly

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Volume 56 March, 1951 No. 1

THE GYRO-FREQUENCY IN THE ARCTIC E-LAYER

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(Received October 30, 1950)

ABSTRACT

Calculation of the gyro-frequency in the E-layer at arctic stations from measurements of critical radio-frequency differences gives a lower magnetic field than that obtained by extrapolation of the terrestrial field measured at ground-level, and, at one station, a large semi-diurnal variation with maxima at 06^h and 18^h local time.

Ray-path deflections which can explain similar effects previously reported for the F-layer cannot be responsible in the E-layer because of the opposite sense of this deviation and because of the small layer-thickness. Moreover, no semi-diurnal variation was found in the F-layer.

It is shown that these new E-layer phenomena may be due to a variable concentration of heavy ions rising to over 4,000 times the density of free electrons.

INTRODUCTION

Measurements of critical radio-frequency differences have been made from the photographic height-frequency records obtained at a number of Canadian ionospheric recording stations (see Fig. 1). Results so far reported have been confined to the F-region.

Calculations of the gyro-frequency from these measurements have revealed interesting phenomena which may be summarized as follows: (a) The calculated

magnetic field in the F-region is usually much too high in comparison to the terrestrial field measured at ground-level and extrapolated to F-region heights; (*b*) large diurnal, seasonal, and other irregular variations in the apparent field occur; (*c*) the field calculated from the longitudinal mode is consistently different and usually lower than the field calculated from the transverse mode.

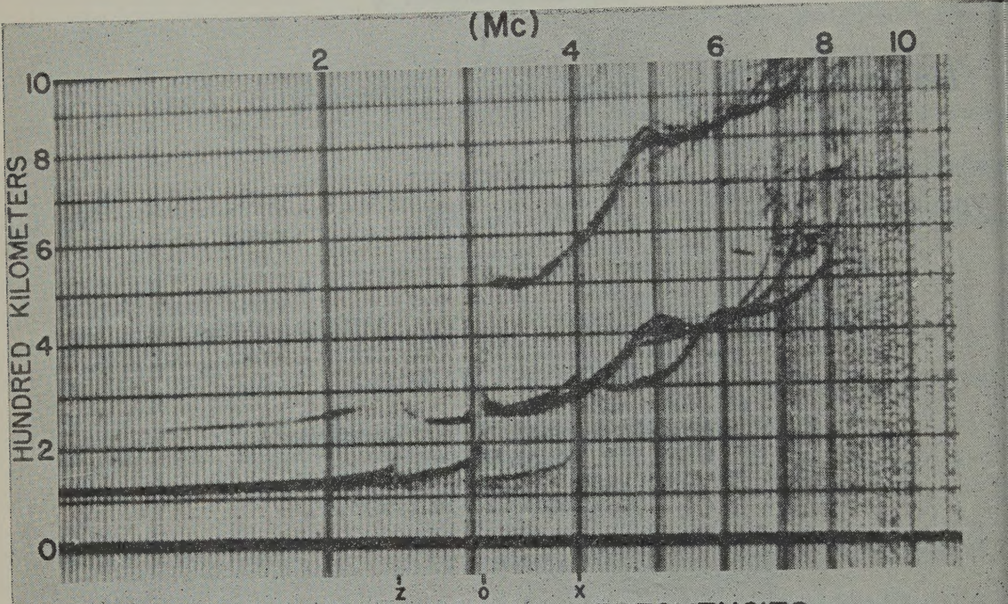


FIG. 1—E-LAYER CRITICAL FREQUENCIES

These F-region phenomena have been explained as due to ray-path deflections coupled with the normal latitude-gradients of ionization [see 1, 2, and 3 of "References" at end of paper].

It has been shown [3] that the ray-path deflections are proportional to the thickness of the layer. Since the thickness of the E-layer is small, such deflections together with normal latitude-gradients of ionization could have only a negligible effect on critical frequencies in this region.

Unless some other factor modifies the E-layer critical frequencies, we would therefore expect calculations of the gyro-frequency from these frequencies to give the true value of the magnetic field of the earth in this layer [4, 5].

Measurements described in detail below show, however, that there are wide divergencies between the gyro-frequency calculated in this manner and the expected field at 100 km. These divergencies are such as might be due to a large and variable concentration of heavy ions [6, 7].

It was for many years considered necessary to postulate considerable quantities of heavy ions in the E-region to explain the current-systems that produce the well-known diurnal variations of the earth's magnetic field [8]. Although the development of tidal theory has reduced this need, it is possible that free electrons alone are still insufficient to account for the observed magnetic variations.

The measurements described in this paper can be satisfactorily explained if we postulate a heavy ion to electron ratio of the order of 4,000. In the absence of any other explanation, these phenomena may therefore be taken as contributary evidence for the presence of these ions.

MATHEMATICAL THEORY OF HEAVY-ION EFFECT

We assume a solution of Maxwell's equations of the form $Ae^{i\omega(sx-t)}$, where $d(sx)/dx=s$ and write the solution in terms of the following parameters:

$$\omega_0^2 = \frac{N_e e_e^2}{\omega^2 m_e}$$

$$\lambda = \frac{e_i N_i}{e_e N_e}$$

$$\mu = \frac{e_e m_i}{e_i m_e}$$

$$g = \frac{\nu_e}{\omega}$$

$$G = \frac{\nu_i}{\omega}$$

$$h = \frac{e_e H_0}{m_e c\omega}$$

In these definitions N_e is the density of electrons of charge e_e and mass m_e, and N_i the density of ions of charge e_i and mass m_i. The angular wave-frequency is ω and the collision frequencies for free electrons and ions, respectively, are ν_e and ν_i. The intensity of the earth's magnetic field is H_0.

Maxwell's equations are then

$$\left.\begin{aligned} \frac{ie_e}{\omega m_e}E_x &= \omega_0^2 V_{ex} + \lambda\omega_0^2 V_{ix} \\ \frac{ie_e}{\omega m_e}E_y &= \frac{1}{1-c^2s^2}(\omega_0^2 V_{ey} + \lambda\omega_0^2 V_{iy}) \\ \frac{ie_e}{\omega m_e}E_z &= \frac{1}{1-c^2s^2}(\omega_0^2 V_{ez} + \lambda\omega_0^2 V_{iz}) \end{aligned}\right\} \quad \text{.........................(1)}$$

and the vector equations of motion for electrons and ions become

$$\frac{ie_e}{\omega m_e}E = (1+ig)V_e - iV_e \times h \quad \text{.............................(2)}$$

$$\frac{ie_e}{\omega m_e}E = \mu(1+iG)V_i - iV_i \times h \quad \text{..........................(3)}$$

The electric field is eliminated from these relations leaving six equations in the components of electron and ion velocity.

For compatibility the determinant of the coefficients of these equations must vanish.

This determinant will not be written out. However, if we neglect collision, the condition for reflection is that the refractive index $cs = 0$.

Imposing these restrictions, the determinant reduces to

$$\begin{vmatrix} 1 & -\mu & 0 & 0 & ih_T & -ih_T \\ 0 & 0 & 1 & -\mu & -ih_L & ih_L \\ -ih_T & ih_T & ih_L & -ih_L & 1 & -\mu \\ 1-\omega_0^2 & -\omega_0^2\lambda & 0 & 0 & ih_T & 0 \\ 0 & 0 & 1-\omega_0^2 & -\omega_0^2\lambda & -ih_L & 0 \\ -ih_T & 0 & ih_L & 0 & 1-\omega_0^2 & -\omega_0^2\lambda \end{vmatrix} = 0 \quad\ldots\ldots(4)$$

In this equation, h_L and h_T are the normalized longitudinal and transverse components of the gyro-frequency for electrons, where the coordinates have been so chosen that the direction of wave propagation is x and the magnetic field is in the xy plane.

After some reduction, this determinant is found to have the following solutions:

$$\omega_0^2 = \frac{\mu}{\mu + \lambda} \quad\ldots\ldots(5)$$

and

$$a\omega_0^4 + b\omega_0^2 + c = 0 \quad\ldots\ldots(6)$$

where

$$a = -(\mu^2 - h^2) - 2(\mu - h^2)\lambda - (1 - h^2)\lambda^2$$

$$b = 2(\mu^2 - h^2) + 2(1 - h^2)\mu\lambda$$

$$c = -(\mu^2 - h^2)(1 - h^2)$$

Since $m_i \gg m_e$, we have $\mu \gg 1$.

Also above the gyro-frequency, for electrons $h < 1$. Consequently, except for very low frequencies, $\mu \gg h$.

Under these conditions, we have very closely

$$\left.\begin{aligned} a &= h^2\lambda^2 - (\mu + \lambda)^2 \\ b &= 2\mu(\lambda(1 - h^2) + \mu) \\ c &= -\mu^2(1 - h^2) \end{aligned}\right\} \quad\ldots\ldots(7)$$

Using the definition of ω_0, we introduce the critical reflection frequencies, $f = f_e/\omega_0$, where

$$f_e^2 = \frac{N_e e_e^2}{4\pi^2 m_e}$$

is the well-known expression for the ordinary mode critical frequency in the absence of heavy ions.

For the ordinary mode, we have $f \equiv f_0$ and from (5)

$$f_o^2 = f_e^2\left(1 + \frac{\lambda}{\mu}\right) \quad \text{(8)}$$

For the extraordinary and longitudinal ordinary modes, by introducing the coefficients from (7) into (6), we obtain a cubic equation in f^2, as follows:

$$\mu^2 f^6 - [\mu^2 f_H^2 + 2\mu^2 f_e^2 + 2\mu\lambda f_e^2] f^4$$

$$+ [(\mu + \lambda)^2 f_e^4 + 2\mu\lambda f_H^2 f_e^2] f^2 - \lambda^2 f_H^2 f_e^4 = 0 \quad \text{(9)}$$

f_H is the gyro-frequency for electrons, and is defined by

$$f_H = \frac{e_e H_0}{2\pi m_e c}$$

This expression can be greatly simplified by substituting f_o for f_e with the help of (8).

We obtain

$$f_H[f_o^2\lambda - f^2(\mu + \lambda)] = \pm f(f_o^2 - f^2)(\mu + \lambda) \quad \text{(10)}$$

In the special case of $\lambda = 0$, this reduces to the well-known relation for the extraordinary and longitudinal ordinary critical frequencies in a free-electron ionosphere.

$$f_H f = \mp(f_o^2 - f^2) \quad \text{(11)}$$

Since the negative sign in (11) gives the extraordinary mode, it is clear that the corresponding positive sign in (10) should give the extraordinary mode in the presence of heavy ions.

Similarly, the negative sign in (10) should give the longitudinal ordinary mode. Using the subscripts x and z for the extraordinary and longitudinal ordinary modes, respectively, we find

$$f_x^3 - f_H f_x^2 - f_o^2 f_x + f_H f_o^2 \frac{\lambda}{\mu + \lambda} = 0$$

and

$$f_z^3 + f_H f_z^2 - f_o^2 f_z - f_H f_o^2 \frac{\lambda}{\mu + \lambda} = 0 \quad \text{(12)}$$

Solving these equations, it is found that each has one positive real root giving the critical frequencies f_x and f_z.

We now have two equations in the three measurable critical frequencies f_z, f_o, and f_x. By elimination of λ/μ, we have an equation for the electron gyro-frequency in the presence of heavy ions.

$$f_H = (f_x - f_z) + \frac{f_x f_z - f_o^2}{f_x - f_z} \quad \text{.............................(13)}$$

and, by elimination of f_H, we have an equation for λ/μ from which we can obtain the concentration of heavy ions of any given mass.

$$\frac{\lambda}{\mu} = \frac{f_x f_z (f_x f_z - f_o^2)}{(f_o^2 - f_x^2)(f_z^2 - f_o^2)} \quad \text{.................................(14)}$$

In the special case of a free-electron ionosphere, $\lambda/\mu = 0$ and $f_x f_z = f_0^2$, so that the critical frequencies are geometrically spaced and f_H reduces to $f_x - f_z$, as is well known.

Thus, both the heavy-ion concentration and the gyro-frequency can be calculated from simultaneous measurements of f_o, f_x, and f_z.

On the other hand, if the gyro-frequency is assumed to be known, as for example by an inverse-cube extrapolation of the measured surface-field to the height of reflection in the ionosphere, then we can calculate the heavy-ion concentration from measurements of any pair of critical frequencies. For this purpose, we obtain the equations

$$\lambda/\mu = \frac{f_x[f_H f_x - (f_x^2 - f_o^2)]}{(f_x - f_H)(f_x^2 - f_o^2)} \quad \text{....................................(15a)}$$

$$\lambda/\mu = \frac{f_z[f_H f_z - (f_o^2 - f_z^2)]}{(f_z + f_H)(f_o^2 - f_z^2)} \quad \text{....................................(15b)}$$

$$\lambda/\mu = \frac{f_x f_z}{f_x - f_z} \, \frac{f_H + f_z - f_x}{f_x f_z - f_H(f_H + f_z - f_x)} \quad \text{.....................(15c)}$$

These relations are illustrated in Figures 2 and 3.

In Figure 2, the frequency intervals $f_z - f_o$ and $f_x - f_o$ are plotted as functions of the ordinary critical frequency f_o for different values of the parameter λ.

The curves were plotted for a gyro-frequency of 1.62 Mc, which is the value to be expected at a height of 100 km at Baker Lake when using the measured intensity of the earth's field at the surface and extrapolating by the inverse-cube approximation.

The value of μ used was 4.33×10^4, which corresponds to an ion mass of 24. This value is arbitrary, since no satisfactory photochemical process has been proposed to account for the presence of large quantities of heavy ions in the ionosphere. However, as μ enters the equations only in the ratio λ/μ, the critical-frequency separations can easily be obtained for other values of ion mass. It is perhaps necessary to point out that the polarity of the ions has no effect on the equations, because in the ratio λ/μ charge enters as e^2.

In Figure 3 the critical frequencies are plotted as functions of the ion concentration for a fixed ordinary mode critical frequency of 3.0 Mc, a gyro-frequency of 1.62 Mc, and an ion mass of 24.

A logarithmic frequency-scale was used to show the departure of the critical frequencies from geometrical spacing as λ is increased.

Also plotted are the corresponding curves of f_e and f_i. Just as f_e is the ordinary

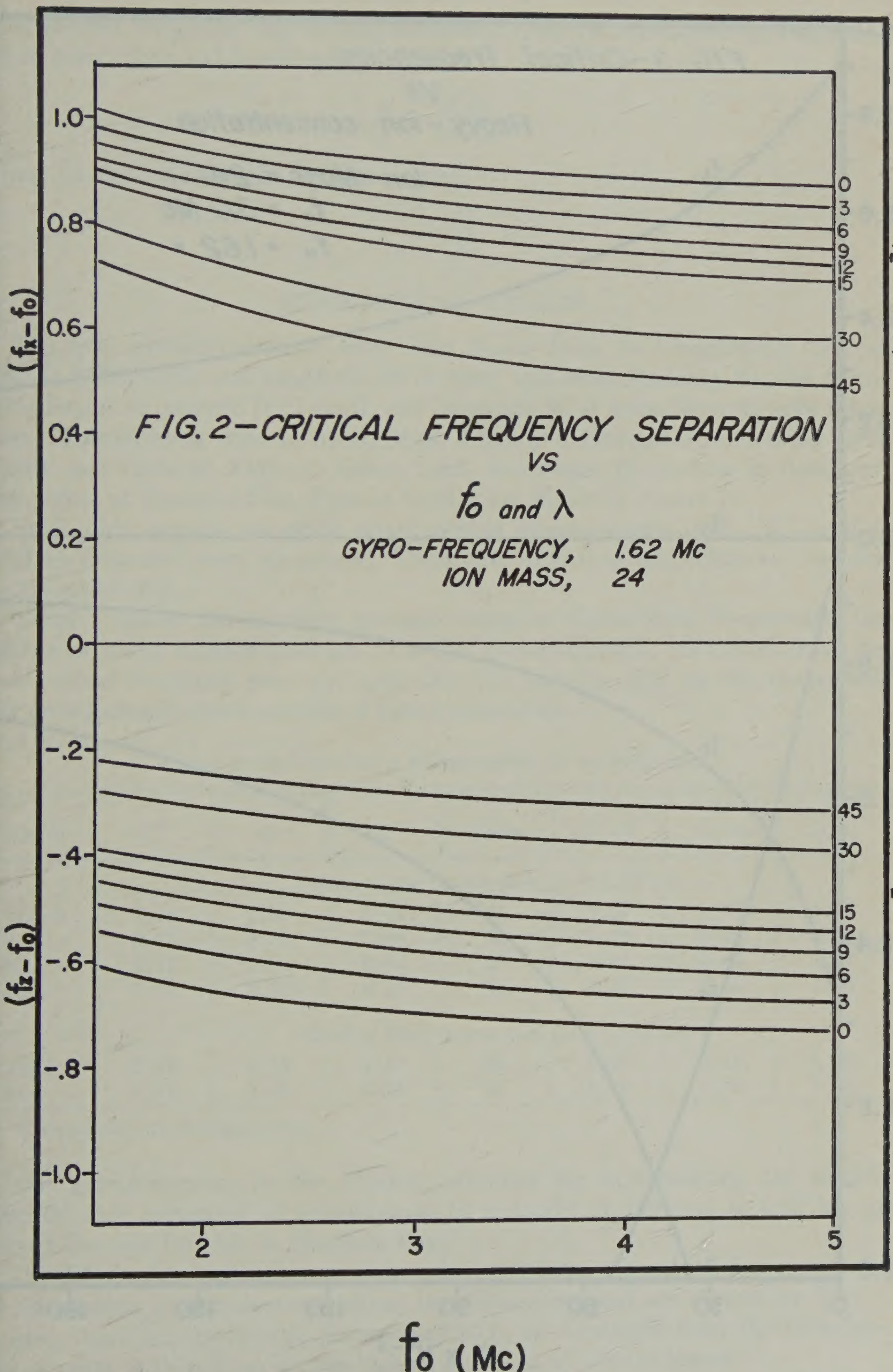


FIG. 2—CRITICAL FREQUENCY SEPARATION
VS
f_0 and λ
GYRO-FREQUENCY, 1.62 Mc
ION MASS, 24
$(f_x - f_0)$
$(f_z - f_0)$
$(\lambda \times 10^{-3})$
1.0
0.8
0.6
0.4
0.2
0
-.2
-.4
-.6
-.8
-1.0
0
3
6
9
12
15
30
45
2
3
4
5
f_0 (Mc)

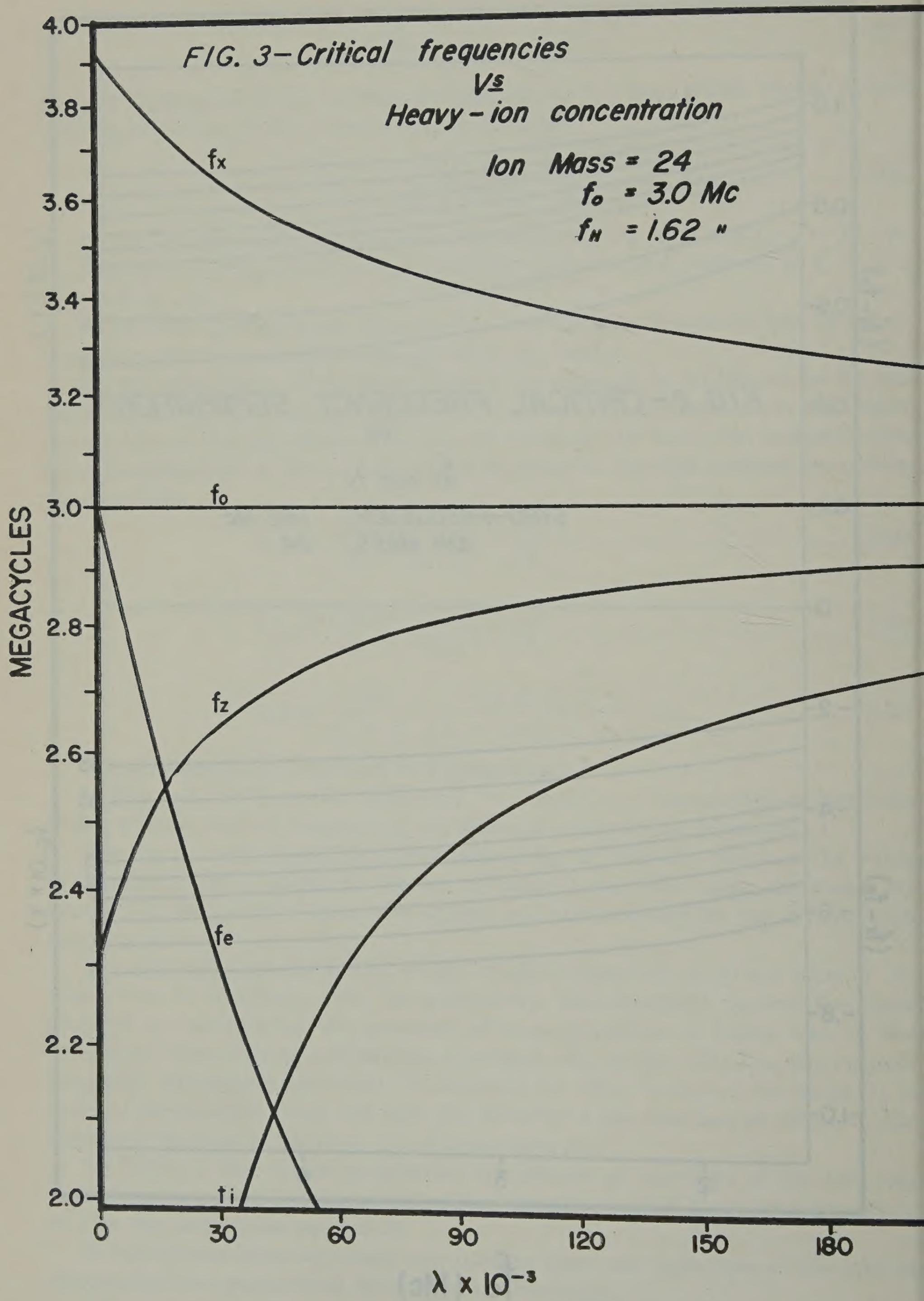


FIG. 3—Critical frequencies
Vs
Heavy-ion concentration
Ion Mass = 24
f_o = 3.0 Mc
f_H = 1.62 "
f_x
f_o
f_z
f_e
f_i
MEGACYCLES
4.0
3.8
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
0
30
60
90
120
150
180
λ x 10⁻³

mode critical frequency due to free electrons, f_i is the corresponding frequency due to heavy ions and is defined by

$$f_i^2 = \frac{N_i e_i^2}{4\pi^2 m_i}$$

It may be verified from the definitions that

$$\frac{\lambda}{\mu} = \frac{f_i^2}{f_e^2}$$

EXPERIMENTAL RESULTS

The first records measured were from Baker Lake on Chesterfield Inlet at latitude 64°.3 north and longitude 96°.0 west, and from Resolute Bay on Cornwallis Island at latitude 74°.7 north and longitude 94°.9 west. Records were taken every 15 minutes in March, April, May, June, and November, and every hour in July and October, 1949, at Baker Lake, and every 15 minutes in April and May, 1949, at Resolute Bay. Typical records are shown in Figure 1.

First, only records on which simultaneous measurements of f_z , f_o , and f_x could be measured were considered. These are, unfortunately, relatively few due to absorption of f_x .

Table 1 shows the monthly average values of these three frequencies, the number of cases in each average, and the gyro-frequency, calculated from the three critical-frequency pairs (f_z , f_o), (f_o , f_x), and (f_z , f_x), on the assumption that no significant concentration of heavy ions exists.

TABLE 1—*Gyro-frequency on assumption of no heavy ions*

Month	f_z*	f_o	f_x	Cases	f_H (z, o)	f_H (o, x)	f_H (z, x)
	Baker Lake; inverse cube f_H = 1.62 Mc						
March	2.14	2.80	3.60	21	1.53	1.43	1.47
April	2.19	2.85	3.68	44	1.52	1.48	1.49
May	2.17	2.84	3.64	23	1.56	1.42	1.47
June	2.39	3.07	3.91	26	1.56	1.43	1.46
	Resolute Bay; inverse cube f_H = 1.57 Mc						
April	2.03	2.68	3.47	33	1.50	1.41	1.44
May	1.91	2.54	3.33	30	1.48	1.39	1.42

*Frequencies are in megacycles.

The gyro-frequency in the E-layer, obtained by extrapolating the earth's magnetic field measured at ground-level to a height of 100 km, is 1.62 Mc at Baker Lake and 1.57 Mc at Resolute Bay.

This Table shows that, when f_H is calculated from the critical frequencies on the assumption of free electrons alone, the values obtained are always too low.

Also, there is a systematic discrepancy in f_H as calculated from the three frequency pairs, with $f_H(z, o)$ always highest and $f_H(o, x)$ always lowest.

These results are just the opposite of those already reported for the F-region.

In the F-region, f_H calculated from the critical frequencies is usually too high and the (o, x) pair usually gives a higher value than the (z, x) pair.

The explanation given for these phenomena in the F-layer in terms of ray-path deflections in the thick F-region and the normal gradients of ionization with

Table 2—*Gyro-frequency and heavy-ion ratio from three critical frequencies*

Month	Average frequency difference $(f_x - f_z)$	Gyro-frequency (f_H)	Ion ratio (λ)
	Baker Lake; inverse cube f_H = 1.62 Mc		
March	1.47	1.38	−2,630
April	1.49	1.46	− 890
May	1.47	1.35	−3,530
June	1.52	1.46	−1,610
	Resolute Bay; inverse cube f_H = 1.57 Mc		
April	1.44	1.37	−2,240
May	1.42	1.34	−2,300

latitude cannot be applied to the E-layer. Not only because in the thin E-layer the deflections must be small, but also because normal latitude-gradients will increase the apparent gyro-frequency, while in the E-region, on the contrary, the apparent gyro-frequency is found to be too low.

Now, if we admit the possibility of heavy ions in the E-region, we can apply equations (13) and (14) to calculate both the gyro-frequency and the ion concentration from the simultaneous measurements of f_z, f_o, and f_x.

Unfortunately, when we do this, we obtain even lower values of the gyro-frequency and negative (and consequently impossible) values of the ion concentration. This is shown in Table 2, for which an ion mass of 24 has been assumed.

The average frequency differences $(f_x - f_z)$ have also been included in Table 2.

Table 3—*Correction to f_x from measured f_z and f_o and inverse-cube field*

Month	f_z*	f_o	f_x		Per cent correction
			Measured	Calculated	
		Baker Lake; inverse cube f_H = 1.62 Mc			
March	2.14	2.80	3.60	3.70	+2.8
April	2.19	2.85	3.68	3.75	+1.7
May	2.17	2.84	3.64	3.75	+3.0
June	2.39	3.07	3.91	3.97	+1.6
		Resolute Bay; inverse cube f_H = 1.57 Mc			
April	2.03	2.68	3.47	3.56	+2.5
May	1.91	2.54	3.33	3.42	+2.9

*Frequencies in megacycles.

It is seen that this frequency difference is always larger than the calculated gyro-frequency.

Examination of equations (13) and (14) and of Figure 2 shows that the negative

values of λ are due to f_o being greater than the geometric mean of f_z and f_x . If heavy ions were present, the gyro-frequency would be greater than the frequency difference $f_x - f_z$, and f_o would be less than the geometric mean of f_z and f_x .

We conclude that the measured simultaneous intervals $f_o - f_z$ and $f_x - f_o$ cannot be explained as due to the presence of heavy ions.

However, another factor must be considered. In the *E*-layer at these latitudes, many more records can be measured for f_z and f_o than for f_x . This is due to the

TABLE 4—*Heavy-ion ratio assuming inverse-cube field*

Month	No. of cases	Ion ratio (λ)	f_o	Simultaneous cases		
				No. of cases	Ion ratio (λ)	f_o
			Mc			*Mc*
			Baker Lake; inverse cube f_H = 1.62 Mc			
March	103	1,230	2.921	21	1,470	2.80
April	123	1,930	2.954	44	1,700	2.85
May	305	3,340	3.089	23	1,060	2.84
June	363	2,310	3.160	26	1,040	3.07
July	96	2,040	3.147	0		
			Resolute Bay; inverse cube f_H = 1.57 Mc			
April	131	680	2.800	33	1,170	2.68
May	118	1,970	2.750	30	1,490	2.54

high absorption of the extraordinary mode. With the present equipment, the critical frequencies can be measured only to the nearest tenth of a megacycle. The significance of the averages used in the calculations depends on the number of measurements included, providing the errors of measurement are random.

However, because of the high absorption of the extraordinary mode, not only are there fewer measurements, but it is quite possible that a systematic error is introduced due to fading out of the record near the critical frequency.

Consequently, we tentatively assume that the monthly average measurements of the interval $(f_o - f_z)$ are correct and assume the gyro-frequency in the *E*-layer to be properly given by the inverse-cube extrapolation of the measured surface-field to a height of 100 km.

The correction to f_x necessary to make it consistent with f_z and f_o on this basis is given in Table 3.

The average measured extraordinary critical frequency is thus between 1.5 to 3.0 per cent below a value consistent with the other two critical frequencies and the extrapolated gyro-frequency. It is believed that a systematic error of this magnitude is possible.

On these assumptions, the heavy-ion concentration is calculated by equation (15*b*) from the monthly average intervals $(f_o - f_z)$, for all measurable records and for records on which f_x was also measured.

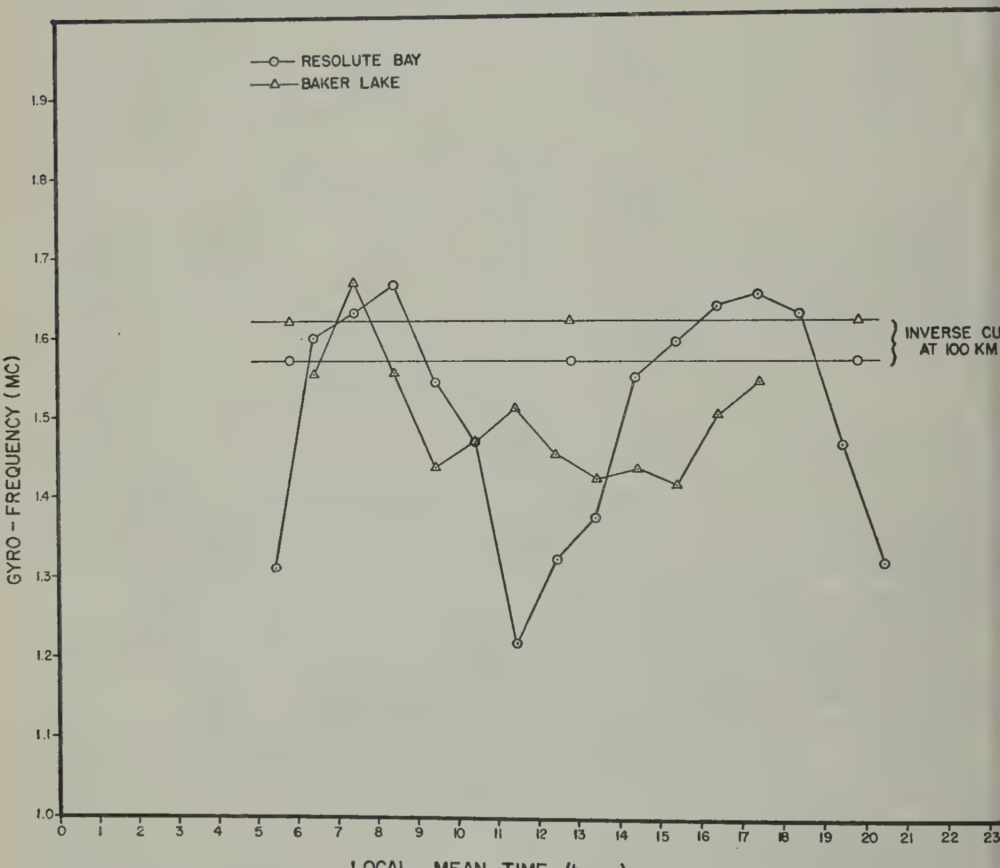
Table 4 gives the resulting values of λ for an assumed ion mass of 24, the average ordinary mode critical frequency, and the number of cases in each average.

It will be noticed that the average f_o when f_x is absorbed is always higher than when f_x is measurable. This is because absorption and the critical frequencies are both highest around noon, so that simultaneous measurements of f_z , f_o , and f_x are usually early morning or late afternoon measurements.

The monthly average ratio of heavy ions to free electrons is seen to lie between 1,040 and 1,700 when the extraordinary critical is measurable, but between 680 and 3,340 when it is usually absorbed.

FIG. 4

APPARENT GYRO-FREQUENCY VARIATION IN THE ABSENCE OF HEAVY IONS



The relative constancy of λ when absorption is low prompts us to examine the diurnal variation of the heavy-ion ratio or alternatively of the apparent gyro-frequency if heavy ions are absent.

The diurnal variation of mean hourly gyro-frequency in the absence of heavy ions is shown in Figure 4, averaged at Baker Lake from March to August, inclusive, and at Resolute Bay for April and May.

It is seen that if heavy ions are absent, unless some other explanation is forthcoming, we must assume that the magnetic intensity in the E-layer goes through

very large diurnal variations amounting to over 20 per cent of the total field. The intensity is generally lower than the inverse-cube field and shows a minimum in the middle of the day and maxima in the morning and evening. These results are, in general, opposite to those found in the *F*-layer and already reported.

Variations and deviations of magnetic intensity of this magnitude are quite incompatible with surface measurements of the terrestrial field, nor is there any plausible source of the large currents required to produce such magnetic effects.

Consequently, we turn to heavy ions as a possible explanation of these phenomena. Plotted in Figure 5 is the heavy-ion ratio derived from the data used in

FIG. 5—RATIO OF HEAVY IONS TO FREE ELECTRONS

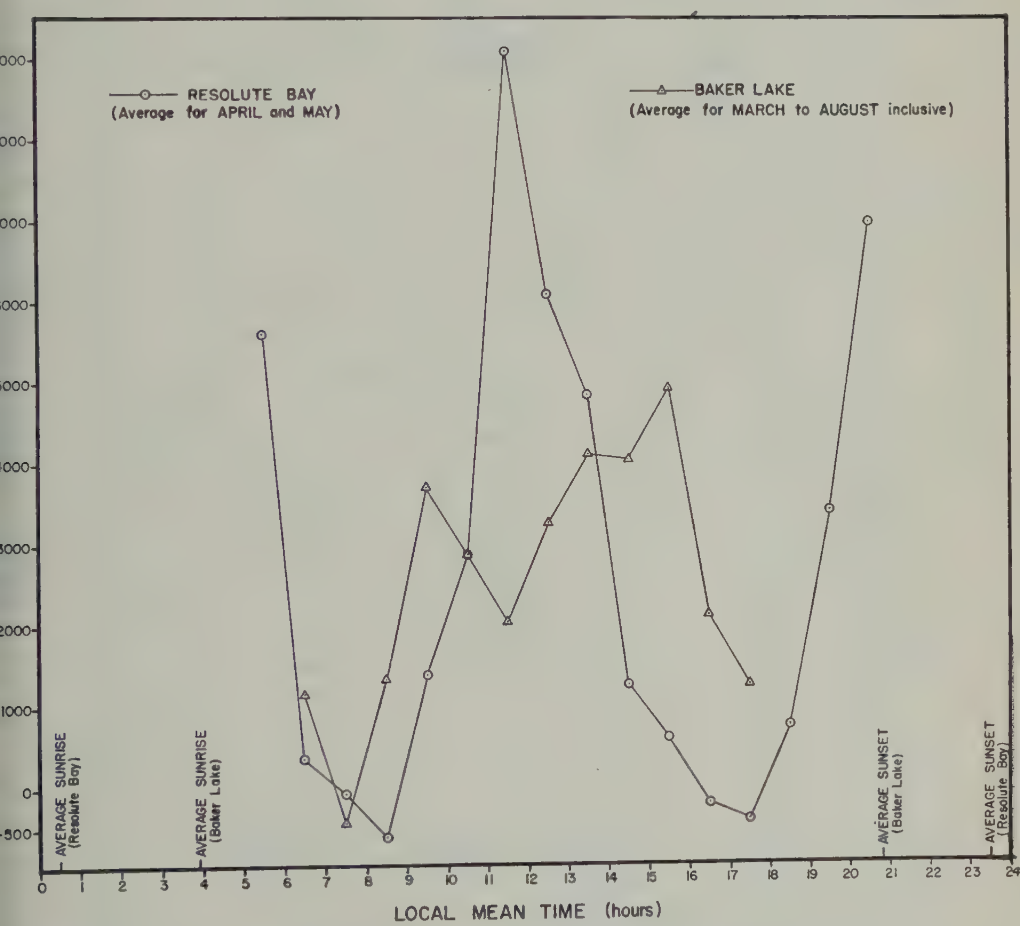


Figure 4, on the assumption that the gyro-frequency has the constant value consistent with surface measurements of the terrestrial field. As would be expected from the curves of Figure 4, the ratio of heavy ions to free electrons shows a diurnal variation with maximum in the middle of the day, and minima morning and evening in which the ion concentration becomes negligible. The average ratio of heavy ion to free electrons is found to be of the order of one to four thousand.

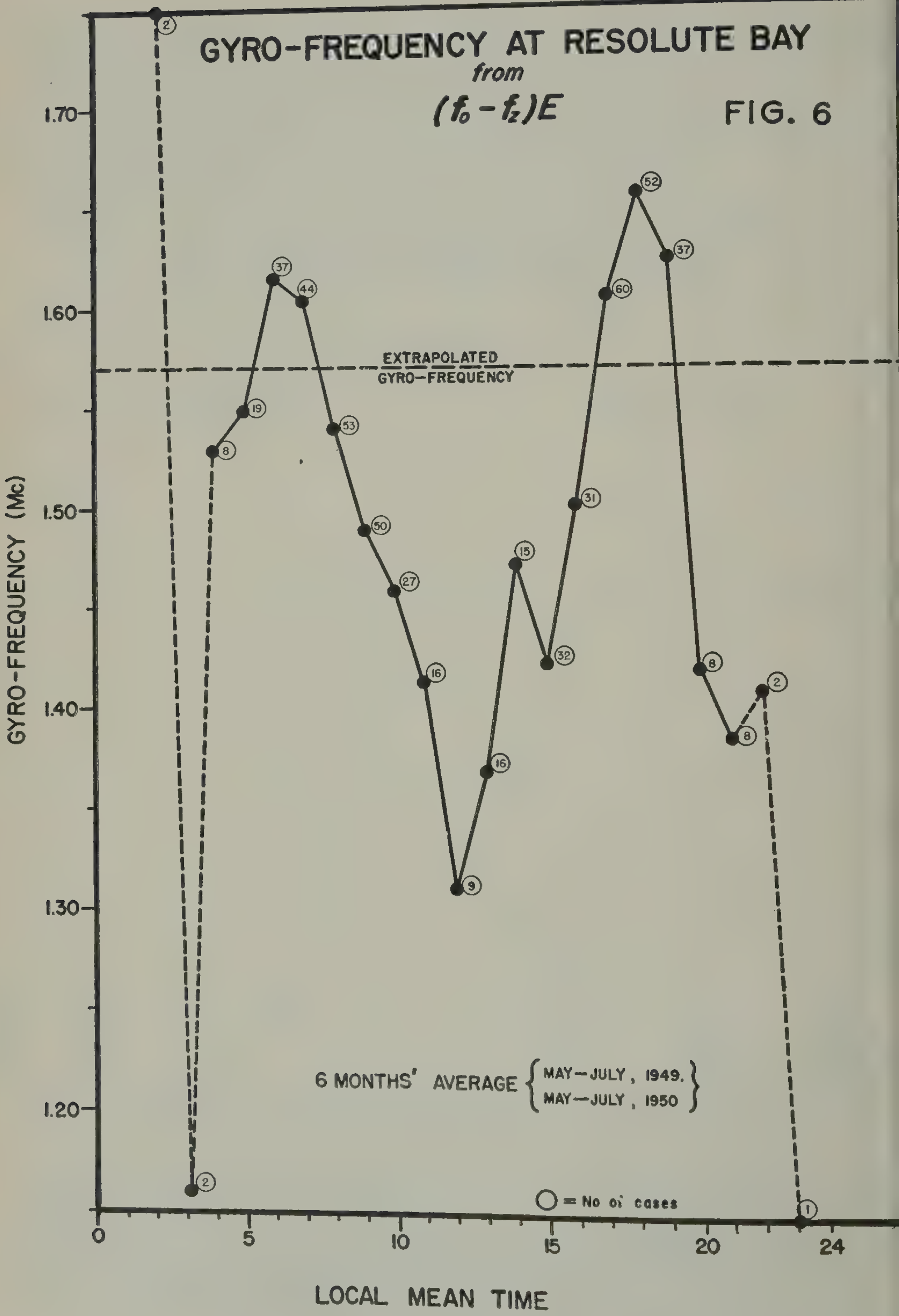
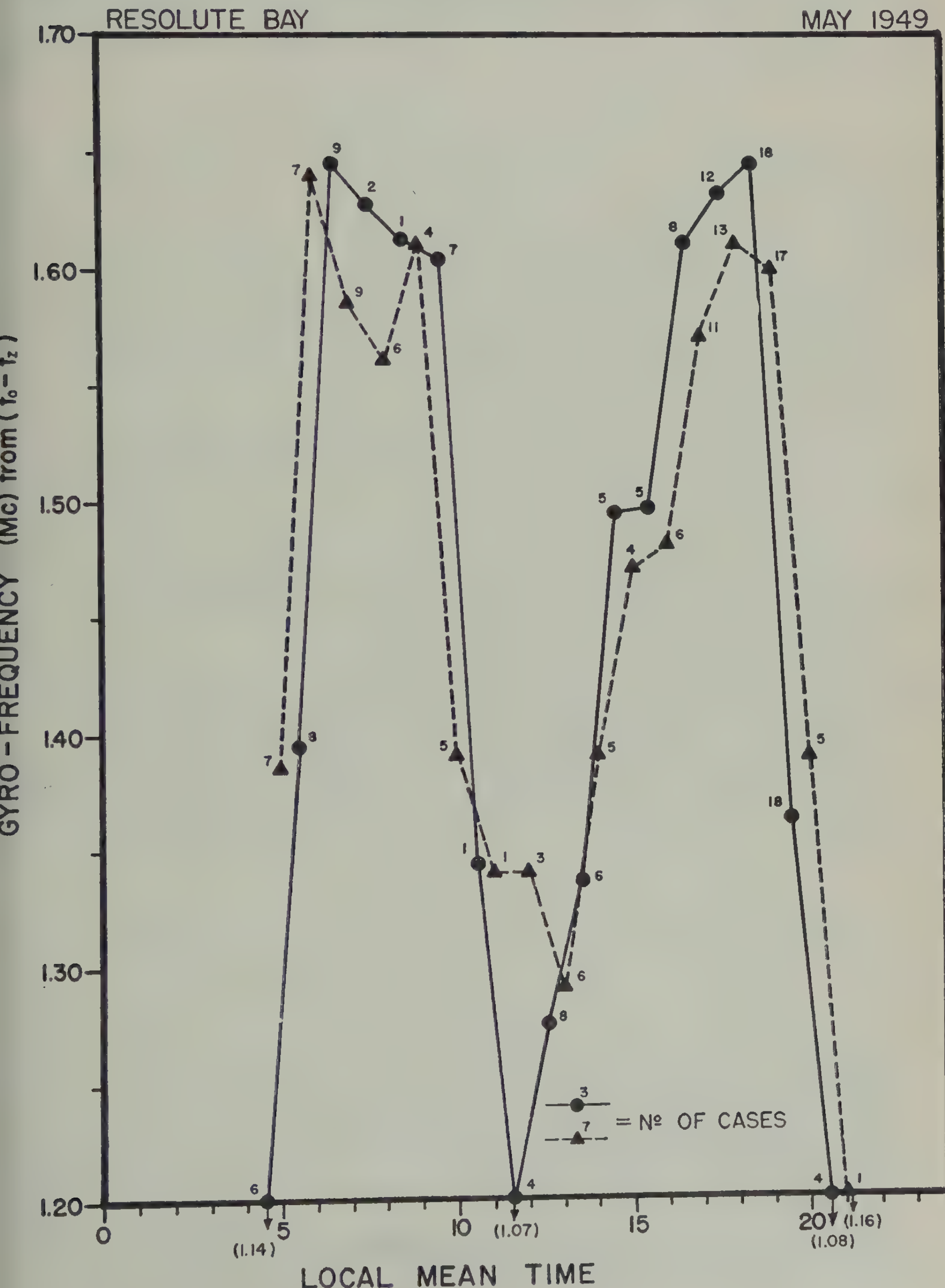

GYRO-FREQUENCY AT RESOLUTE BAY
from
$(f_o - f_z)E$
FIG. 6
EXTRAPOLATED GYRO-FREQUENCY
GYRO-FREQUENCY (Mc)
1.70
1.60
1.50
1.40
1.30
1.20
6 MONTHS' AVERAGE { MAY—JULY, 1949. MAY—JULY, 1950 }
○ = No of cases
0
5
10
15
20
24
LOCAL MEAN TIME

FIG. 7

COMPARISON OF INDEPENDENT MEASUREMENTS IN E-LAYER



The rejection of measurements of the extraordinary critical frequency on the grounds of possible systematic error is not altogether satisfying. However, it is clear from Table 1 that, if the heavy-ion ratio had been calculated from either the *ox* or *zx* pairs according to equations (15*a*) or (15*c*), even higher heavy-ion densities would have been found.

SEMI-DIURNAL VARIATION AT RESOLUTE BAY

The semi-diurnal variation at Resolute Bay indicated in Figures 4 and 5 was further examined in May to July, 1949 and 1950.

This provided six months of arctic day records taken every 15 minutes. The records for May, 1950, were again scaled to provide an independent check of the previous measurements included in Figures 4 and 5.

The gyro-frequency calculated from the hourly average $(o - z)$ E-layer critical frequencies is shown in Figure 6. The number of measurements included in each six-month average is shown on the Figure, and it will be seen that the curve near midnight is insufficiently established.

However, there can be no doubt about the reality of the maxima at 06^h and 18^h local time and the noon minimum. This variation is firmly established by the data for the separate months. In every month, the maxima at about 06^h and 18^h hours and the noon minimum are evident.

The reliability of the measurements is indicated in Figure 7. The records for May 1949 were scaled twice, quite independently. It is interesting that, although considerably fewer records were treated as measurable on the second run, the main features of the variation are repeated closely.

In addition to the large semi-diurnal variation, there is some evidence for another peak at about 14^h of more variable amplitude. However, further data will be required to establish the significance of this subsidiary maximum.

Whether these semi-diurnal variations in the apparent gyro-frequency are due to an inverse variation of the heavy-ion ratio or reflect a real variation in the local magnetic field, it seems likely that the explanation will be found in tidal phenomena.

ACKNOWLEDGMENTS

I am indebted to Mr. David V. Dickson for scaling the photographic records, computing the curves, and calculating the tabulated gyro-frequencies.

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CHARACTERISTICS OF THE E_s REGION AT BRISBANE

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ABSTRACT

The E_s region as recorded on routine $h'f$ records taken at Brisbane (latitude 27°.5 south, longitude 153°.0 east) between June 1943 and December 1949 has been studied. At all seasons the critical frequency is lowest at dawn. In summer months, the critical frequency reaches a maximum at about 10^h and then declines gradually, remaining high until after midnight. In winter, the rise is slower and the maximum critical frequency occurs around 14^h, dropping markedly by sunset. In general, a high E_s critical frequency is accompanied by blanketing of F echoes up to a comparatively high frequency; but, whereas the occurrence of high critical frequencies shows a summer maximum and a winter sub-maximum, the occurrence of high blanketing frequencies is least frequent in winter. The observations suggest that there are two distinct types of E_s common at Brisbane—one formed at greater heights and descending to its final position, the other formed *in situ*. The first, the predominant type in summer, blankets strongly and has probably a uniform ionization density; but the second, the winter type, blankets little and probably has pronounced lateral irregularities. No evidence of correlation could be found with sunspot numbers, ionospheric storms, or meteor occurrence frequency, and the conclusion is reached that the Brisbane E_s is not predominantly of meteor origin. There is some slight evidence of correlation between the constant-height type of E_s and F region diffuseness.

I—INTRODUCTION

The E_s region, which is an abnormal or sporadic region occurring in the ionosphere at heights between 100 and 200 km, differs from the normal regions in that no group retardation is observed when the maximum frequency it reflects is approached. Moreover, in general, it is penetrable by radio waves of frequency appreciably less than this maximum frequency. These features suggest that the ionization gradients at the boundaries of the region must be very steep, the region itself being either very thin (maximum thickness of the order of a few hundred metres), or very patchy in horizontal distribution.

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Surveys of occurrence of E_s in various parts of the world have been published [see 1, 2, 3, 4, and 5 of "References" at end of paper]. The results obtained at Watheroo (latitude 30°.3 south, longitude 115°.9 east) and at other Australian and New Zealand stations would be expected to be most closely comparable with those at Brisbane on account of the comparative proximity of these stations.

II—TECHNICAL DETAILS

II.1—*Equipment*—Two separate automatic equipments have been in use between 1943 and the present time, the original recorder being replaced in July 1947. The two recorders were not greatly different, the first one having a power output of approximately 200 watts in a 100-200 microsecond pulse, while the latter gives approximately 500 watts in a 50-100 microsecond pulse. In general, the usable receiver sensitivity is limited by the site noise level and is therefore more or less independent of the particular receiver in use. The new recorder has an extended low frequency range, going down to 1 Mc/sec, as opposed to the 2.2 Mc/sec lower limit of the old recorder. Due to variations in radiated power, receiver sensitivity, and errors in receiver tuning, the over-all sensitivity of the recorder is by no means constant throughout the band of frequencies covered (1-16 Mc/sec).

When the new recorder was installed, the aerial system was changed over from a pair of Berkner aerials (large diameter, centre fed, horizontal aerials, 132 feet long over-all, 40 feet above ground) to a pair of delta aerials at right-angles, using an 80-foot pole as centre support. Both the Berkner and the delta systems give predominantly upward radiation patterns over most of the frequency range, but the patterns differ in detail and undoubtedly side lobes of considerable importance exist at certain frequencies. Hence the echoes seen on the records do not necessarily come from that part of the ionosphere immediately above the equipment. At the same time, consideration of the theoretical polar diagrams of the aerial systems shows that at no frequency in the working range is there a null in the upward direction.

II.2—*Scaling of E_s characteristics*—The specification of the total amount of E_s ionization present at any recording station presents a problem. In this report we work in terms of the quantities fE_s and fbE_s, that is, the "critical frequency" and the "blanketing frequency," defined respectively as the highest frequency at which E_s echoes are recorded, and the lowest frequency at which echoes are received from layers above the E_s. The definition of fE_s is applicable, and the recorded value is independent of transmitter power only when the recorded echo cuts off sharply as frequency increases. One type of E_s layer has a very sharp critical frequency, the reflection coefficient falling from a comparatively high value to zero within a few tenths of a megacycle. The critical frequency of this type can be read to 0.1 or 0.2 Mc/sec. However, another type of E_s region is often observed whose reflection coefficient decreases slowly as the frequency of the exploring wave increases. In this case, the highest frequency giving a recorded echo will depend on the sensitivity of the system, and the end of the trace is sometimes hard to determine. The policy at Brisbane is to read to the end of the continuous portion of the trace (disregarding gaps due to known recorder faults). The critical frequency under such circumstances may be in error by perhaps

1 Mc/sec. For both types of E_s the virtual height can be read with an accuracy of 10 km.

Rawer [6] has proposed a more elaborate method of measurement, which involves a comparison of E_s and F echo strengths. It requires measuring equipment rather more elaborate than is available at Brisbane.

When it is possible to distinguish the ordinary and extraordinary rays separately, the critical frequency of the ordinary ray is read as fE_s. When the ordinary and extraordinary rays overlap, the highest recorded frequency is scaled.

III—TEMPORAL VARIATIONS OF E_s CHARACTERISTICS

III.1—*Diurnal variations of critical frequency*—Although some E_s is nearly always present at Brisbane, its critical frequency shows a marked diurnal variation. The curves of Figure 1 show the monthly average of hourly values of fE_s for representative months of 1949. Where no E_s could be seen, the value of fE_s has been taken as zero. Such curves have been plotted for each month for several years. The curves reproduced represent only a small sample of the data available, but an examination of the remainder shows that Figure 1 is fully representative of the average behaviour.

The most characteristic feature of the diurnal variation at all seasons is a definite minimum at dawn. Following this, the critical frequency rises to a maximum, rapidly in summer, more slowly in winter. The maximum occurs at about 10^h in summer and about 14^h in winter. At the equinoxes there is some evidence of a double maximum, with a shallow minimum at about noon. The subsequent decline in critical frequency is very gradual in summer, not becoming appreciable until after midnight, whereas in winter a rapid decline sets in at about sunset. At the equinoxes there are comparatively long periods each day for which no E_s reflection can be discerned in the records.

III.2—*Diurnal variations of blanketing*—An examination of the amount of blanketing of F-region echoes by the E_s region indicates a general positive correlation between the blanketing frequency and the E_s critical reflection frequency. Curves of the monthly average of hourly values of fbE_s (corresponding to the curves of fE_s in section III.1 preceding) are shown in Figure 2. The curve for June is probably not indicative of the absolute amount of blanketing in winter, since at that season the blanketing frequency is difficult to read accurately, the $h'f$ record at such times often having an appearance as shown in Figure 3. In the analysis on which Figure 2 was based this has been regarded as indicating an E_s region blanketing up to the point of bifurcation. But it is equally, if not more, likely that the low frequency portion of the trace is due, in part at least, to reflections from the normal $E1$ region, and that there is no blanketing at all. All that one can decide with certainty from such records is that fbE_s is less than f^oE1.

The curves of Figure 2 indicate that the existence of a high blanketing frequency is confined to the daytime in winter, but persists into the night in summer.

It is considered that the E_s region behaves with sufficient regularity for one to be able to assume that there is a good correlation between the average value of fE_s or fbE_s and the number of occasions during any given period when fE_s or fbE_s exceeds some arbitrary frequency. Consequently, we can get a comparative

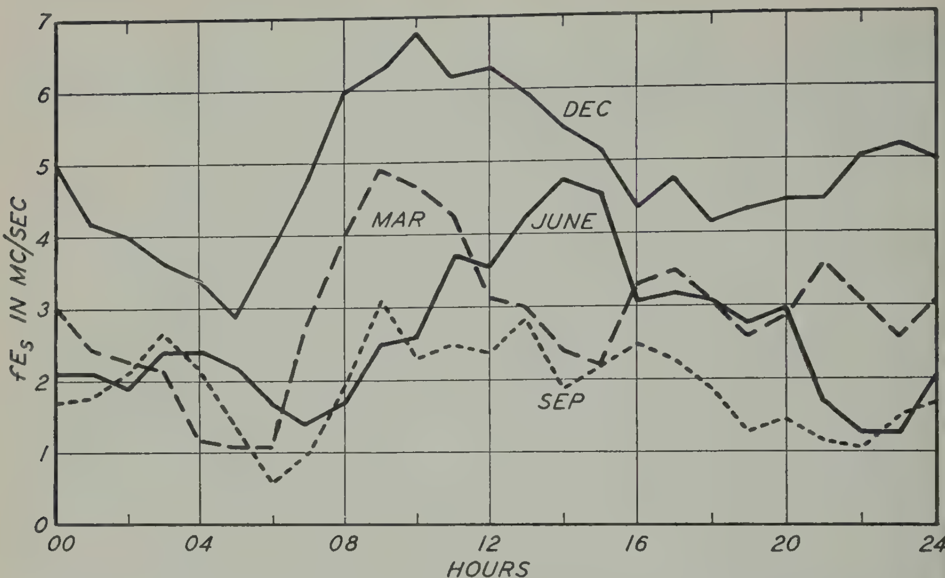

FIG. 1—DIURNAL VARIATIONS OF CRITICAL FREQUENCY; MEAN HOURLY VALUES OF fE_S FOR MARCH, JUNE, SEPTEMBER, AND DECEMBER, 1949

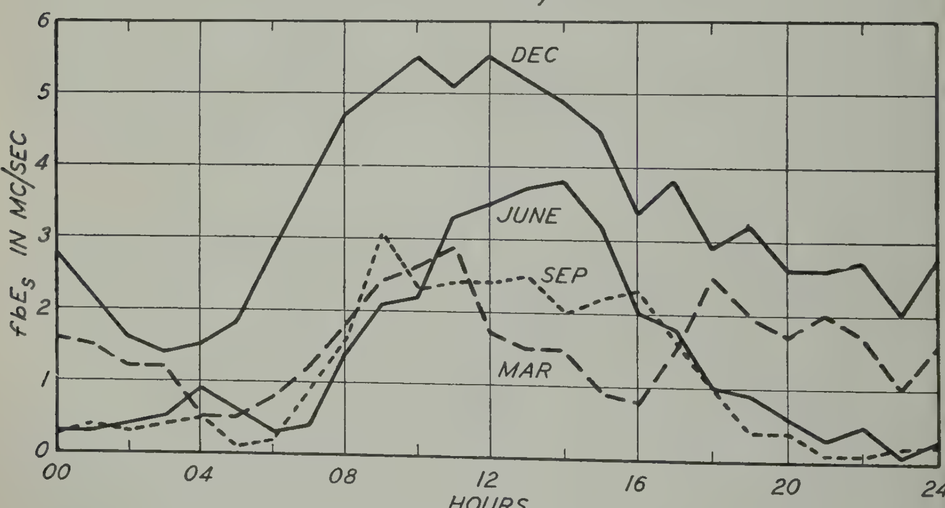

FIG. 2—DIURNAL VARIATIONS OF BLANKETING; MEAN HOURLY VALUES OF fbE_S FOR MARCH, JUNE, SEPTEMBER, AND DECEMBER, 1949

picture of the blanketing behaviour by plotting the number of hours at the various seasons for which the blanketing frequency exceeded 4 Mc/sec. These curves, which are not reproduced, are in good agreement with those of Figure 2. They also show that in some years there is some slight evidence of a double maximum of blanketing during the daytime in summer, with a shallow dip around noon.

III.3—*Seasonal variations of critical frequency*—Following the assumption of positive correlation between average values and number of occurrences of high values (made in the last paragraph of the preceding section), an analysis has been made of the number of hours in each month for which the critical frequency as measured at the beginning of each hour exceeded 7 Mc/sec, for the period from

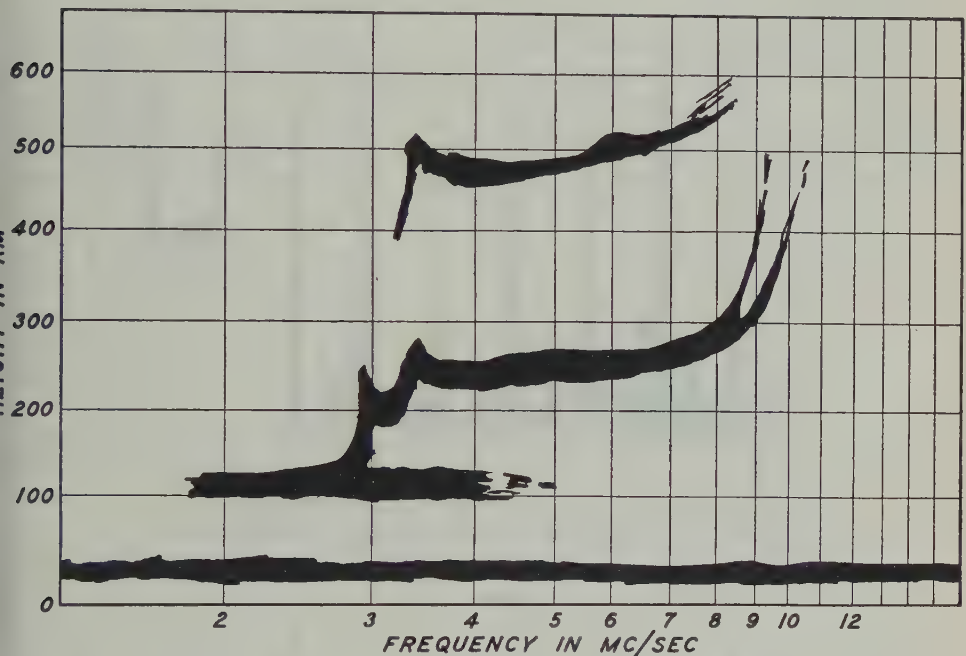

FIG. 3—DOUBTFUL VALUE OF THE BLANKETING FREQUENCY; TRACING OF THE h′f RECORD FOR 14^{h}40^m JUNE 7, 1950 (THE BLANKETING FREQUENCY ON SUCH A RECORD CANNOT BE READ ACCURATELY BECAUSE OF THE OVERLAPPING OF THE E_S AND E1 TRACES)

June 1943 to December 1949. The results of this analysis are shown in Figure 4. Similar analyses were made for critical frequencies exceeding 5 Mc/sec and 3 Mc/sec. The histograms so obtained are similar in general appearance to those for 7 Mc/sec. They all show that:

(*a*) The critical frequency most frequently reaches very high values, and therefore the average critical frequency is highest, during summer. This is indicated by the pronounced rise shown for the months of December and January each year.

(*b*) The critical frequency least frequently reaches high values, and therefore the average critical frequency is lowest, around the spring equinox (September 21). In two years of the series, the minimum value is at the autumn equinox, but the average values show a deeper minimum in spring.

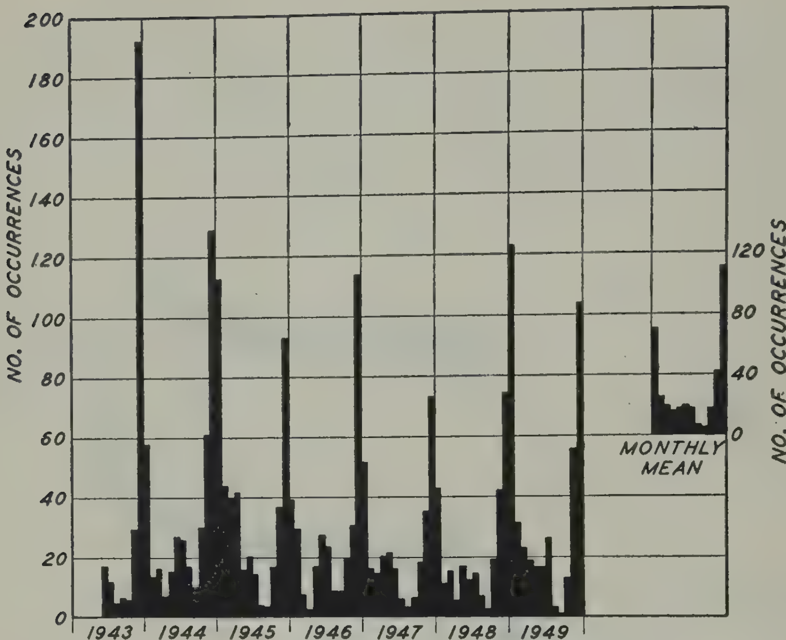

FIG. 4—SEASONAL VARIATIONS OF CRITICAL FREQUENCY; HISTOGRAM OF THE NUMBER OF HOURLY RECORDS PER MONTH ON WHICH fE_S EXCEEDED 7 MC/SEC, FOR THE PERIOD JUNE 1943 TO DECEMBER 1949

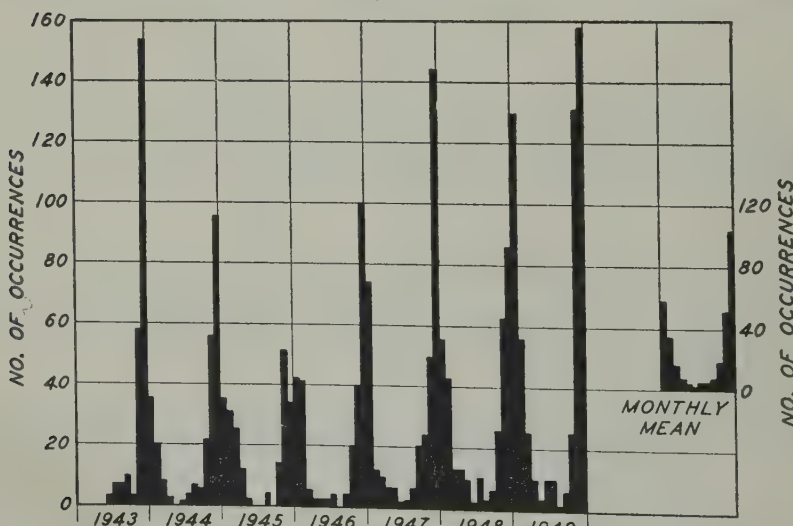

FIG. 5—SEASONAL VARIATIONS OF BLANKETING; HISTOGRAM OF THE NUMBER OF HOURLY RECORDS PER MONTH ON WHICH fbE_S EXCEEDED 5 MC/SEC, FOR THE PERIOD JUNE 1943 TO DECEMBER 1949

(*c*) Compared with the behaviour at the equinoxes, there is an increase in occurrence of high critical frequencies, and therefore of the average critical frequency, during the winter. Although this is not very marked in the average curve (shown in inset), it is clearly shown for many of the individual years.

III.4—*Seasonal variations of blanketing*—In Figure 5 are plotted the number of hours in each month for which blanketing of F echoes by E_s was observed to extend beyond 5 Mc/sec, as measured at the beginning of each hour, for the period June 1943 to December 1949. The curves all indicate clearly that extensive blanketing of F echoes by E_s reaches a marked peak at the summer solstice; there is a deep minimum at the winter solstice, particularly noticeable on the inset showing

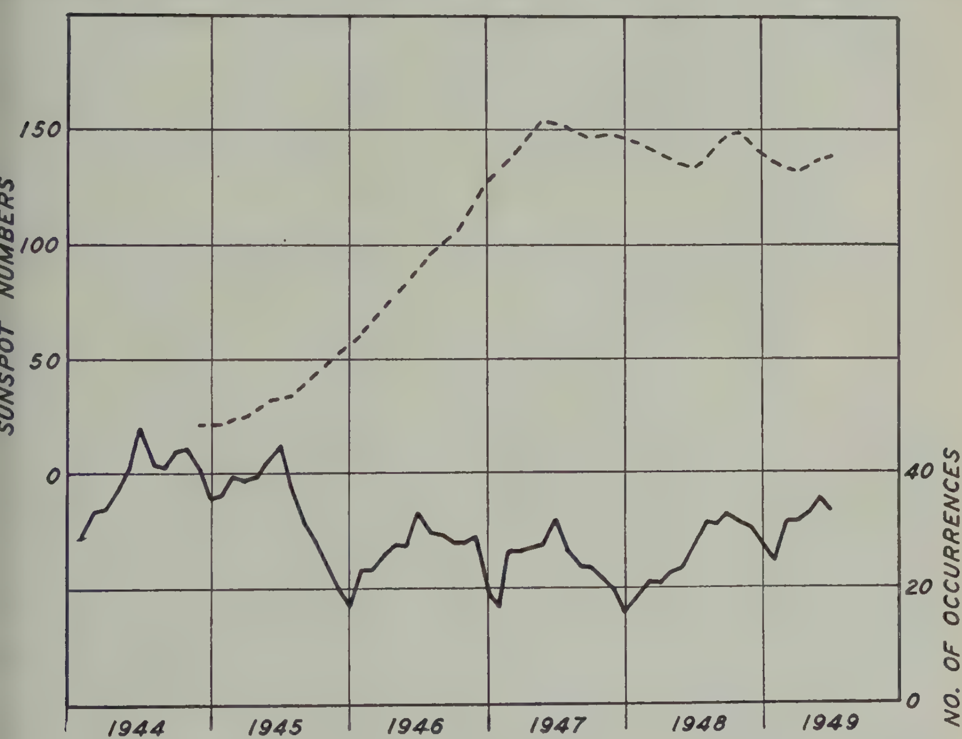

FIG. 6—SUNSPOT-CYCLE VARIATIONS OF fE_S; CURVE OF THE 12-MONTH RUNNING MEAN OF THE NUMBER OF HOURLY RECORDS ON WHICH fE_S EXCEEDED 7 MC/SEC, FOR THE PERIOD DECEMBER 1943 TO DECEMBER 1949, TOGETHER WITH A CURVE OF SMOOTHED RUNNING AVERAGE OF SUNSPOT NUMBERS

the mean over a number of years. This behaviour is in striking contrast to that exhibited by the critical reflection frequency curves discussed in III.3 above. Taken together, these curves indicate that, whereas in the summer-time the in-

creased frequency of occurrence of high critical reflection frequencies is matched by an increased frequency of occurrence of high blanketing frequencies, the second increase of reflection in the winter-time is actually accompanied by decreased blanketing. To check this point, a more extensive search was made for a winter maximum of blanketing, taking lower frequency limits of 3 and 4 Mc/sec, over a number of years, without positive result.

III.5—*Sunspot-cycle variations of critical frequency*—The period of the observations covers a substantial part of a sunspot cycle. The evidence of the Brisbane records in respect to a correlation between E_s behaviour and the phase of the sunspot cycle is somewhat contradictory. One method of searching for such correlation is to plot, for each month during the period, the 12-month running mean of the number of E_s occurrences at frequencies above 7 Mc/sec. The result is shown in Figure 6, together with a plot of smoothed running average of sunspot numbers. The E_s curve shows some evidence of a negative correlation with sunspot numbers. On the other hand, by selecting, in place of 7 Mc/sec, a lower frequency limit, we obtain a curve (not reproduced) which shows a steady rise, amounting to perhaps 25 per cent over-all, during the period of the observations. It seems doubtful, therefore, whether any significance attaches to either result. Other workers [1, 2] have also referred to the lack of any marked sunspot-cycle variation of E_s ionization.

III.6—*Sunspot-cycle variations of blanketing*—Exact figures have not been compiled, but from the appearance of Figure 5, it is clear that there is no marked variation of E_s blanketing throughout the sunspot cycle.

IV—THE TWO TYPES OF E_s

IV.1—*General*—The foregoing analysis of the temporal variations of E_s occurrence shows a discrepancy between the seasonal variation of reflecting behaviour, and the seasonal variation of blanketing behaviour. The most plausible explanation of the discrepancy is that there are, in fact, two types of E_s region—one a heavily blanketing type, strong in summer; and the other a non-blanketing type, strong in winter. Other evidence for this hypothesis is the fact that it is possible, by detailed examination of the $h'f$ records for any particular day, to classify the E_s layers represented on the records into two distinct types, differing in respect to the phenomena associated with their appearance, and in respect to their blanketing properties. For convenience, we have in the remainder of this paper employed for these two types the names E_{ss} (sporadic E, sequential type) and E_{sc} (sporadic E, constant-height type).

In the following paragraphs it is shown that the diurnal and seasonal variations of E_{ss} and E_{sc} are of a different nature. These conclusions have been reached as a result of a detailed examination of each quarter-hour record for four selected months of 1946, taken in conjunction with the routine examination of all of the other records of the 1943-49 period.

IV.2—*The "sequential" type* (E_{ss})—The sequential type originates at a height where little or no other E_s is visible. The first phenomenon associated with its appearance is a stratification of the $E1$ or $F1$ layer leading to the formation of a layer (namely, the $E2$), which may itself be stratified, at about 150-200 km, but

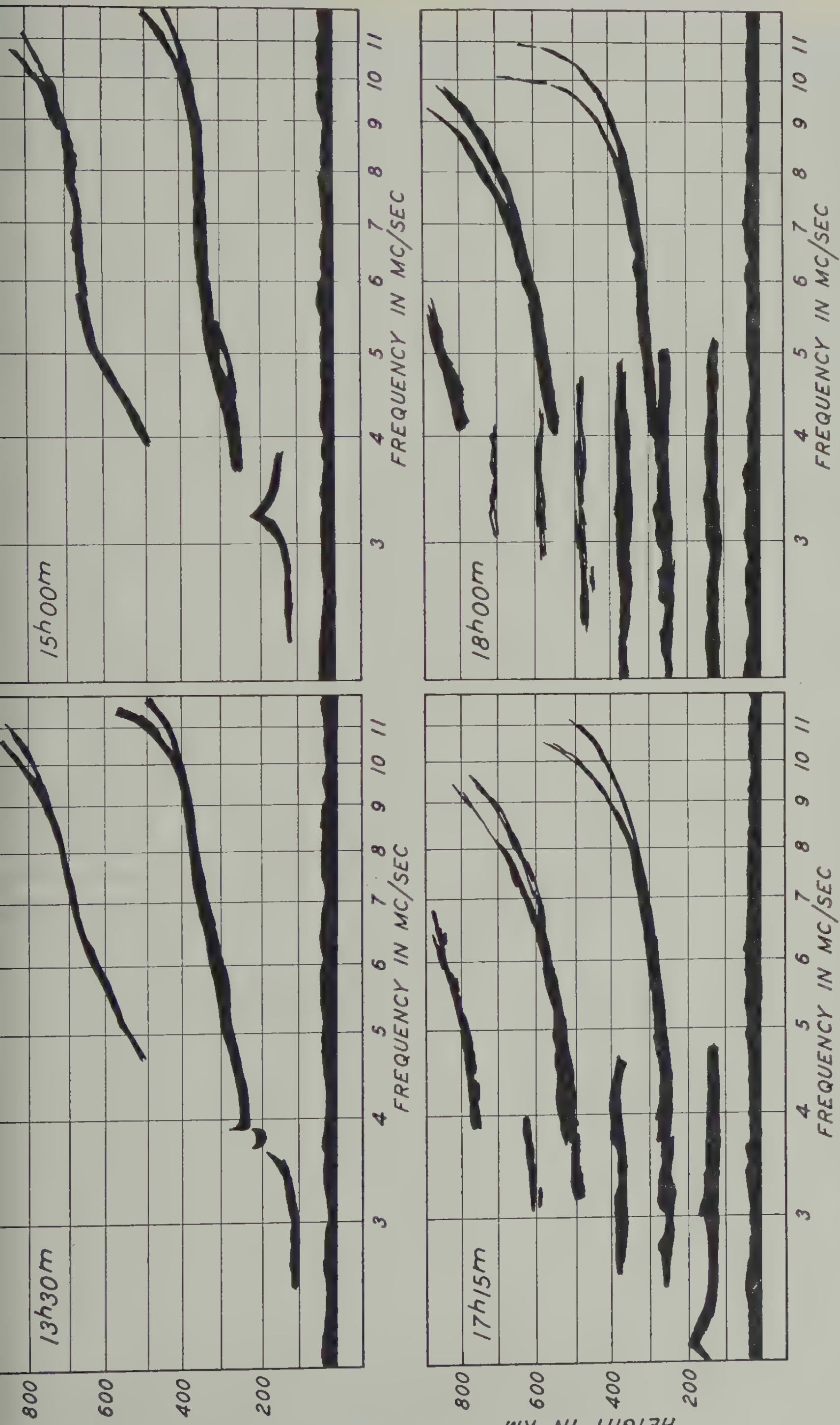

FIG. 7—FORMATION SEQUENCE OF E_{SS} REGION; SERIES OF TRACINGS OF h'f RECORDS FOR FEBRUARY 28, 1947, SHOWING THE REGION COMMENCING TO FORM FROM AN E2 REGION AT 180 KM AND FINISHING AT 110 KM

otherwise having properties similar to those of $E1$ or $F1$. Part of this $E2$ layer then moves slowly downwards, and as it does so it extends its reflecting properties to higher frequencies, losing, in the initial stages of the process, the upward sweep at the penetration frequency. At this stage, the o and x rays show separately. The layer eventually reaches about 100-110 km, by which time the o and x rays have merged almost completely. Until the layer has fallen to 140 km, its blanketing frequency is not detectably less than its critical frequency. Figure 7 shows a series of tracings of $h'f$ records, taken during a summer day, in which such a movement is apparent. This type of formation has been mentioned previously [7, 8], but seems to have received little attention when the origin of E_s ionization is under discussion. The term "sequential" was introduced to indicate an E_s layer formed in such a sequence.

A second characteristic feature of E_{ss} is its strong blanketing of F echoes for almost the whole period of its existence. A typical record of the blanketing is shown in Figure 8. It is only during the last stages in the life of an E_{ss} region which has persisted for some hours that the strong blanketing by the region may cease.

This blanketing feature of E_{ss} carries important implications. A possible explanation of the apparent decrease in height of the layer might be in terms of a horizontal movement of an ionospheric irregularity initially displaced laterally from the recording station. Such an irregularity could, however, have no effect on pulses transmitted vertically from the station to the F layer above it, and thus could not produce the strong blanketing observed in the early part of the formation sequence.

The picture with which we are left is thus of a uniform layer of ionization formed initially at an apparent height of about 180 km, gradually descending and, as it does so, becoming more concentrated, until eventually a thin uniform layer of considerable electron density is formed at about 100-110 km. If this persists for a considerable period, the layer may become patchy in the last stages, instead of disappearing uniformly.

That the E_{ss} layer is usually of very great horizontal extent and uniformity is indicated by a comparison of a number of simultaneous records of $h'f$ recorders at Townsville (latitude 19°.2 south), Brisbane (latitude 27°.5 south), and Canberra (latitude 35°.3 south). These stations have approximately the same longitude and therefore the same local time. Formation sequences appearing on one record appeared also on the others and reached their fully developed stage simultaneously. It was found, however, that the more southerly the station the less clear-cut the sequence, the later the start, and the greater the tendency for decay to intervene before the sequence was complete.

The most frequent hour for commencement of a sequence is immediately after sunrise; but sequences starting soon after midday were observed on many occasions, following morning sequences. It is in fact probable that such afternoon sequences are of more frequent occurrence than the records indicate, for a strongly blanketing E_{ss} formed in the morning would obscure an afternoon formation. Occasionally a "morning" and an "afternoon" E_{ss} are both visible simultaneously on the same record. The tendency for formations to be grouped into mornings and

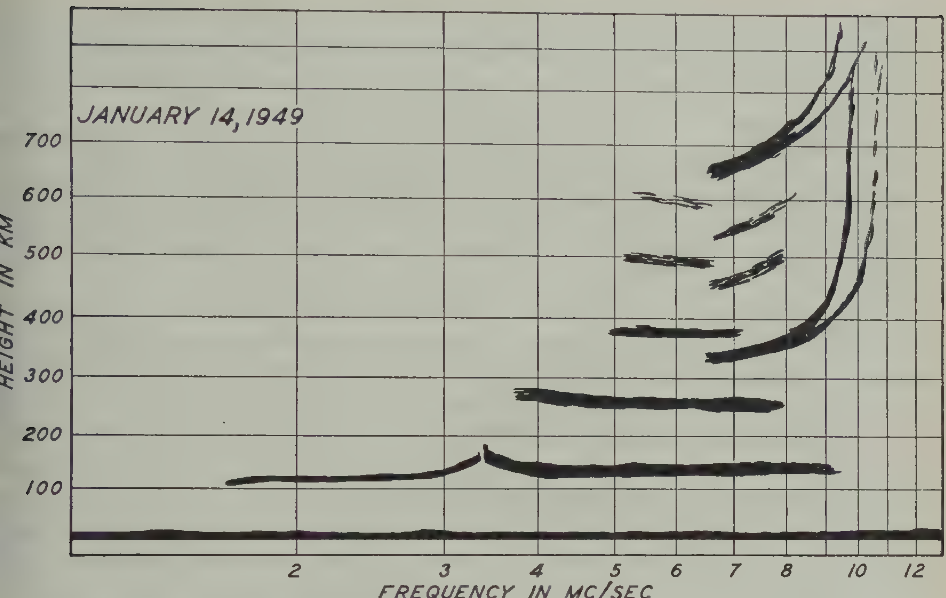

FIG. 8—STRONG BLANKETING BY E_{SS} REGION, FOR $16^h 20^m$

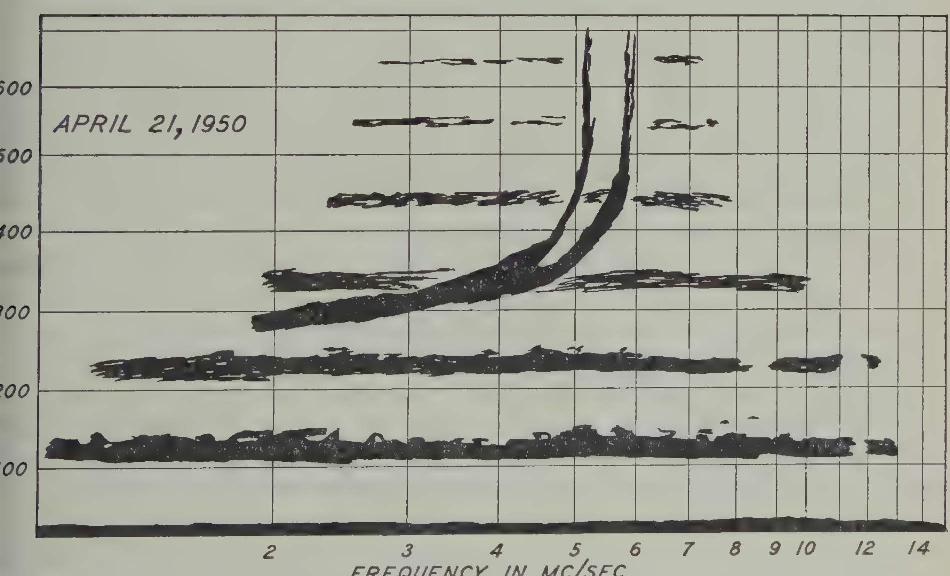

FIG. 9—WEAK BLANKETING BY E_{SC} REGION, FOR $00^h 40^m$

ternoons probably explains the observed diurnal double maximum of blanketing.

The life-time of an E_{ss} region depends on the season and the hour of its formaon. Morning E_{ss} in summer-time sometimes completes its formation sequence in vo hours, commences almost immediately to decay, and completely disappears y noon. Afternoon E_{ss} forms more slowly and the sequence is usually not ac-

complished until after sunset, decay sometimes not being complete until midnight. The diurnal distribution for summer E_s shown in Figure 1 probably represents fairly closely the average diurnal variation of electron density within the E_{ss} region, for in summer this is the predominant region.

E_{ss} is to be regarded essentially as a summer phenomenon, the ionization density to which this mechanism gives rise being greatest at the summer solstice and least at the winter. However, records indicate that sequences occur on one day in four even at midwinter. Brisbane winter-time sequences, like those at Canberra in summer, tend to decay before formation is complete.

It was observed that where two sequences occurred on the one day, the reflection coefficient of the layer was somewhat lower in the second one than in the first. It was also noted that an especially high blanketing frequency for the morning E_{ss} was followed by an especially high blanketing frequency for the afternoon E_{ss}.

IV.3—*The "constant-height" type* (E_{sc})—The second type of E_s layer appears in its final position, usually without any changes of height. It causes very weak blanketing, which never extends beyond 3 Mc/sec, and rarely as far. In spite of this weak blanketing, its reflection coefficient (indicated by multiplicity of echoes) can be quite high (at night), but it is nevertheless lower on the average than that of the E_{ss} layer. A tracing of a typical record is shown in Figure 9.

These facts suggest a picture of the constant-height E_s as a non-uniform region, containing patches of dense ionization embedded in a less densely ionized background. This hypothesis is supported by the fact that, on occasions, a decrease in height has been observed shortly after the region first became visible; while at other times the echo is rather diffuse, or even split into several layers of slightly different apparent height. Both of these effects could be due to appreciable amounts of energy being returned from scattering centres situated some distance to the side of that portion of the ionosphere immediately above the recorder. In the first case, the initial decrease of range could be due to the nearest scattering centre being at first to one side, and gradually moving horizontally until it was overhead.

It is difficult to determine the diurnal and seasonal variations of E_{sc} with any degree of certainty, for unless it appears at a time when E_{ss} is absent, or in an early stage of formation, there will be nothing on the record to mark its appearance.* Further, in summer-time the daytime $E1$ reflection extends to such high frequencies that an E_{sc} layer, unless possessing very great electron density, might be obscured by the normal E. Subject to these uncertainties, the records indicate that there is no marked correlation between occurrence of E_{sc} and the time of day. They suggest a winter maximum and a summer minimum of occurrences; in this respect the evidence is therefore consistent with the conclusions drawn in section IV.1 from the over-all seasonal variation of E_s reflection and E_s blanketing

V—CORRELATIONS

V.1—*General*—It is of some importance to ascertain whether the variations in E_s are related in any way to other ionospheric phenomena. A few investigations have been made in this field.

*It is possible that the apparent night-time relic of a sequential layer may in some cases really be a new layer of E_{sc} type.

V.2—*Ionospheric storms*—Ionospheric storms are well-known phenomena which ollow, after an interval of about one day, certain solar disturbances. A few records ndicated that, preceding an ionospheric storm, the density of E_s ionization is nhanced. A more complete study has shown, however, that this is a relatively are occurrence, which may be purely coincidental.

V.3—*Diffuseness*—There is a certain superficial similarity between the diffuse eflections sometimes accompanying $F2$ reflections, and the E_s reflections, es- ecially from the E_{sc} layer. No correlation between occurrence of diffuseness and f over-all E_s could be detected, but a direct comparison of diffuseness and E_{sc} as not been made. Since diffuseness is most noticeable in winter and at night, it s not impossible that a correlation may exist between these phenomena.

V.4—*Meteors*—Appleton and Naismith [9] have suggested that E_s ionization is rimarily due to meteors. A direct comparison has been made here for only one hort period, *viz.*, during the Giacobinid shower of 1946 [10]. An E_{sc} layer was, in act, formed (Fig. 10), but this lasted only for the duration of the shower and ave the impression of a tenuous, somewhat complex, fluctuating layer. Since this hower was a particularly intense one, it is difficult to account for the strong choes of long duration recorded at other times from E_{sc} .

Comparison of the seasonal variations of E_{sc} with those of meteors provides o evidence of positive correlation. The seasonal maximum of meteors is in the utumn, whereas E_{sc} is less prevalent at that season than at midwinter. On the ther hand, examination of the diurnal variations does provide some slight evi- ence of correlation, insofar as extremely weak E_{sc} echoes frequently appear at round 06^h local time, when meteors are arriving most frequently.

In the case of E_{ss} , correlation with meteors in respect to diurnal and seasonal ariations is, if anything, negative. The diurnal minimum of E_{ss} at sunrise con- rasts with the meteor maximum, and no meteor maximum corresponds to the igh summer peak of E_{ss} . In any case, the systematic formation sequence always ssociated with E_{ss} rules out the possibility of meteoric origin.

VI—DISCUSSION

The well-marked correlation between E_{ss} and the amount of solar radiation, hown by both the diurnal and seasonal variations, suggests that this region is ormed by a fairly direct action of some part of the solar spectrum. The absence f correlation with sunspot-cycle phase indicates, however, that it is unlikely that ltraviolet radiation is responsible. The E_{ss} layer is possibly produced by visible ight through some process such as photodetachment or ionization of pre-excited toms.

The results reported in this paper are definitely unfavourable to any sug- estion that E_{ss} can have a meteor origin. Apart from the negative correlations nentioned in section V.4, it would be difficult to account for the fact that E_{ss} can, t times, have a high reflection coefficient for waves of frequency greater than 2 Mc/sec, whereas only a comparatively feebly-reflecting layer was produced by he intense Giacobinid shower of 1946.

The rather scanty information in respect to E_{sc} provides little, if any, clue to ts origin. It does not rule out the possibility that this type is due to meteors, but

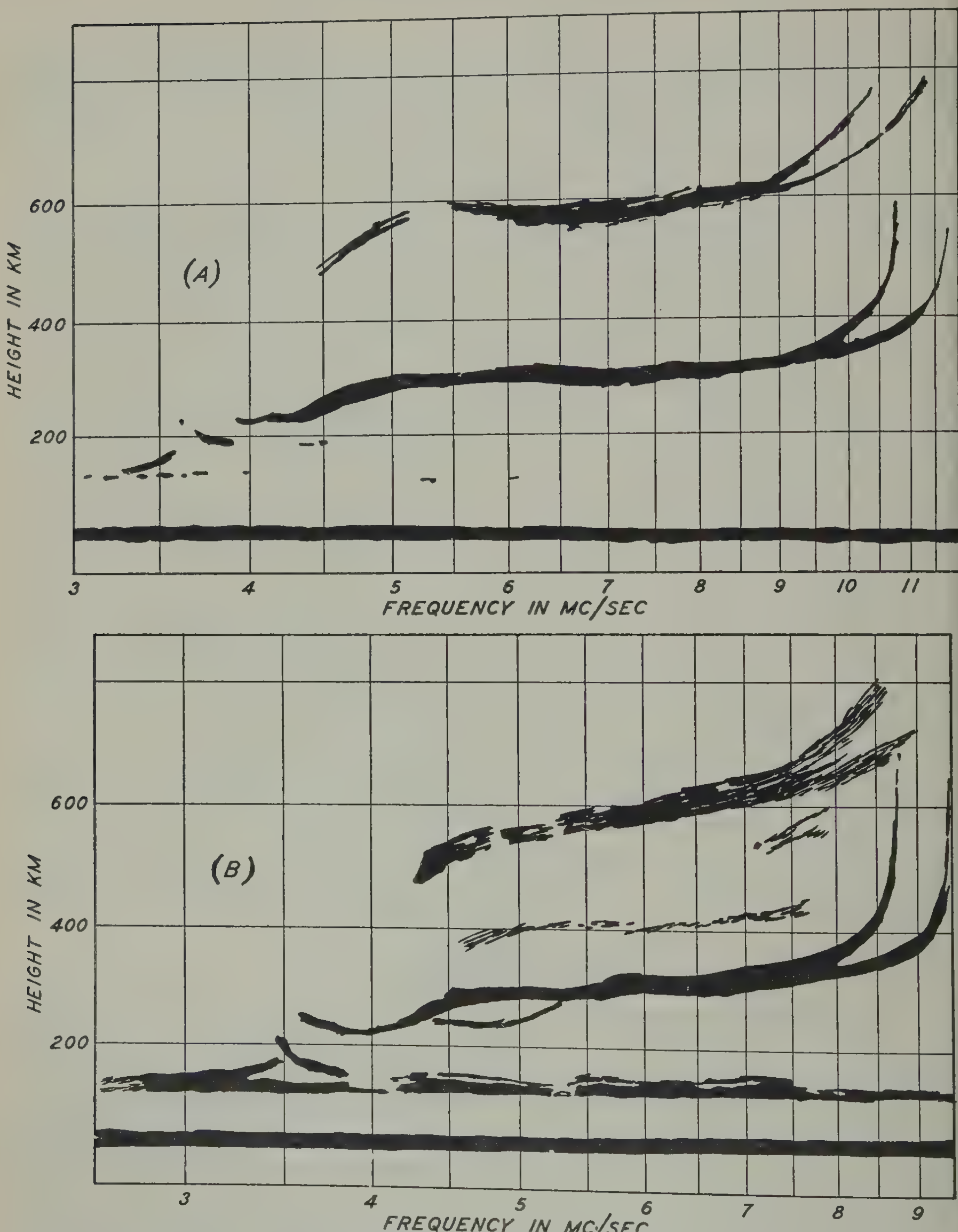

FIG. 10—E_{SC} LAYER FORMED DURING STRONG METEOR SHOWER ON OCTOBER 10, 1946: (A) FIRST INDICATION OF METEOR IONIZATION AT 13^h00^m—TRANSIENT ECHOES AT 100 KM; AND (B) MAXIMUM OF STORM AT 14^h30^m—WELL-DEFINED LAYERS SHOWING AT 100 KM AND ABOVE

it provides no support for that hypothesis. The patchy nature of the layer suggests that its formation is associated with considerable turbulence, and its winter maximum suggests that the mechanism producing it is inhibited by the incidence of sunlight.

VII—ACKNOWLEDGMENTS

The information in this report has been obtained from records of the ionospheric recorder operated by the Physics Department of the University of Queensland in conjunction with the Radio Research Board of the Australian Commonwealth Scientific and Industrial Research Organization, and is published with the permission of the latter body. The authors wish to thank Prof. H. C. Webster for his many helpful suggestions.

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THE APPROXIMATE MEAN HEIGHT OF THE THUNDERCLOUD CHARGES TAKING PART IN A FLASH TO GROUND

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ABSTRACT

A method is described for the measurement of the approximate mean height, H, of the thundercloud charges taking part in a flash to ground. The results obtained from 10 flashes give an over-all mean of 5.2 km, in agreement with those obtained previously by other methods.

(1) *Introduction*—From a comparison of photographic and electrical records of ightning discharges to ground, Malan and Schonland [see 1 of "References" at end of paper] in 1947 found that the duration of the whole leader-process was roughly twice that of the portion photographed after emergence from the cloud-base. As the vertical length of the visible portion was found from direct measurements to vary between 2.0 and 2.5 km, the height from which the discharge came was estimated by them to be from 4.0 to 5.0 km. They found that a study of the variation in form of the leader field-change with distance gave comparable results.

In the experiments to be described, this height has been determined by recording simultaneously the total field-change due to a discharge to ground at three suitably spaced stations. The total electrostatic field-change at any one of these stations is given to a first approximation by

$$E = 2QH/(H^2 + D^2)^{3/2}$$

where H is the vertical length of the discharge, Q the total quantity of electricity lowered by the discharge, and D the horizontal distance from the point of observation. If the three values of D are found by independent location of the flash, H can then be determined from the solution of three simultaneous equations for E.

If, as is usual, the flash consists of more than one stroke, the quantity H is an approximation to the mean of the heights of the separate charges involved in such a multiple discharge.

(2) *Apparatus and method*—Electrostatic fluxmeters, working on the principle of the periodic screening of an exposed electrode as described by Malan and Schonland [2] in 1950, were used to determine E. They were mounted upside down to shield them from rain, on a wooden structure about six feet high.

Three of these instruments were placed at the corners of a triangle about

7 km apart. They were calibrated by comparing simultaneous measurements of the electrostatic field in open country, using the stretched-wire method, and of the fluxmeter-output. The sensitivity of each fluxmeter was found to be about 90 volts per metre per volt output after amplification.

The outputs from the fluxmeters were transmitted by radio to the Bernard Price Institute, where they were simultaneously recorded on a multi-channel galvanometer-recorder. The transmission involved a sub-carrier which fully amplitude-modulated a transmitter operating in the 7-metre band. The output from each fluxmeter, after amplification and rectification, varied the frequency of the sub-carrier. At the Institute, the sub-carrier was converted back into the equivalent of the original rectified fluxmeter-output and recorded by a galvanometer with a period of 0.025 sec. The relative amplifications of the three channels from fluxmeter-output to recording galvanometers were determined by calibration with a signal generator.

The bearings and distances of the individual discharges to ground were determined by direct observation from the Institute. These observations were recorded by code on the field-change record. The bearing accuracy was about 30 degrees, and the distances were obtained from the lightning-thunder interval.

(3) *Results*—The results obtained for 10 discharges which were known to have passed from cloud to ground are given in Table 1 below. The distance of each discharge as observed from the Institute was taken to be correct, and its approximate bearing was adjusted so that the adjusted distances of the discharge from the other stations satisfied the three simultaneous equations for the quantity H.

Table 1—*Results of* 10 *discharges known to have passed from cloud to ground*

Flash No. (1)	Distance from Institute (2)	H (3)	Q (4)	$2QH$ (5)	Remarks (6)
	km	*km*	*coulombs*	*coulomb-km*	
1	8.5	5.3	12.3	130	Bearing adjusted 20°
2	6.5	3.2	12.3	78	Bearing correct
3	4.7	5.0	40.2	402	Bearing adjusted 30°
4	4.7	5.9	17.7	209	Bearing adjusted 30°
5	6.0	4.5	4.5	41	Bearing adjusted 30°
6	4.3	8.7	28.5	495	Bearing adjusted 10°
7	9.8	2.5	29.9	149	Bearing adjusted 20°
8	8.5	6.4	12.3	157	Bearing adjusted* 120°
9	6.5	3.0	16.4	98	Bearing correct
10	4.8	7.8	4.2	66	Bearing adjusted 10°

*Wrong bearing.

Column (3) shows that the values of H vary from 2.5 to 8.7 km, with an average of 5.2 km. The electric moment $2QH$ of the discharge varies from 41 to 495 coulomb-km, with an average of 182.5 coulomb-km. Values for $2QH$ of the same order have

en obtained previously by Schonland and Craib [3, 1927] in the district of merset East, Cape Province, who found that the mean value of $2QH$ for 82 scharges was 94 coulomb-km.

(4) *Acknowledgments*—Thanks are expressed to Dr. B. F. J. Schonland at hose suggestion this work was undertaken, and to Dr. P. G. Gane, Dr. D. J. alan, Mr. J. H. Stephen, and other members of staff of the Bernard Price Initute who have given greatly valued assistance. The fluxmeters were constructed Mr. J. A. Keiller.

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THE INTERPRETATION OF RADAR ECHOES FROM METEOR TRAILS

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ABSTRACT

Electromagnetic-wave theory is applied to various cylindrically symmetric distributions of electron density to ascertain the dependence of reflected radio signals upon the system parameters. Results are expressed in terms of the average effective dielectric constant, the circumference/wave-length ratio of the cylinder, and the incident wave polarization. Two models have been mainly considered—a linear radial variation of electron density, and a constant density distribution. The general properties and dependences of the echoes are summarized for each major region of trail parameters, and the transitional characteristics are indicated.

A qualitative comparison of these results with published observational data leads to a new picture of the trail decay period.

I—*Introduction*

The past decade has witnessed the introduction of radar techniques in the study of meteoric phenomena. The characteristic types of echoes obtained vary widely both in duration and in fine structure, posing many questions which have not as yet received a satisfactory answer. The interpretation of these received signals in terms of the trail characteristics requires a comparison of the experimental results with theoretical calculations based on appropriate physical models. The theory to be found in the literature at present [see 1 of "References" at end of paper] considers only the limiting case of low electron densities confined in cylinders of small diameter, a combination of parameters which appears to preclude its application to most detectable meteor trails at the wave frequencies currently employed. It is the purpose of the present paper to ascertain the behaviour to be expected of the received signal over a range of parameters which one may reasonably expect to encounter in practice. The work is carried through for several models, to determine the degree of dependence of the calculated results on the specific distribution assumed.

II—*Basic methods employed*

The analysis of the scattering of electromagnetic waves by a column of electrons proceeds by formulating Maxwell's equations for the system; the effect of the free charge may be represented, through its equation of motion, by an equivalent complex conductivity or reduced dielectric constant. The latter choice is made here, since it is consistent with the terminology employed in the past literature. The equations for the field components are then (see Appendix for this and future details)

$$\nabla^2\mathbf{E} - \nabla(\nabla\cdot\mathbf{E}) + k^2K\mathbf{E} = 0 \quad (1)$$

$$\nabla^2\mathbf{H} - (\nabla\times\mathbf{H})\times\frac{\nabla K}{K} + k^2K\mathbf{H} = 0 \quad (2)$$

where

$\mathbf{E}$ = electric field strength (mks units)
$\mathbf{H}$ = magnetic field strength
k = wave number $(2\pi/\lambda)$
K = effective dielectric constant $\left(1 - \frac{Ne^2}{\epsilon_0 m\omega^2 u}\right)$
N = electron density
e = electron charge
m = electron mass
ϵ_0 = permittivity of free space
ω = angular frequency of wave
$u = 1 + \frac{i\nu}{\omega}$; ν = electron collision frequency

For the case of normal incidence, one employs equation (1) or (2), depending, respectively, on whether the electric or magnetic vector of the incident wave lies parallel to the cylinder axis. To proceed further, a specific distribution function must be chosen for the electron density, N. Two models have been mainly considered: a linear radial variation from a maximum on the axis to zero at the cylinder surface, producing any average density, N_0; and a constant density distribution. The latter model, of course, leads to the well-known Bessel functions of argument $k\sqrt{K}r$ for the field magnitudes. For the linear variation, we write $K = sr + t$, where s and t can conveniently be expressed as

$$s = \frac{3}{a}(1 - K_0), \qquad t = 3K_0 - 2$$

a is the radius of the cylinder and K_0 is the average effective dielectric constant. Separation of variables then leads to the equations

$$\frac{d^2T_m}{dr^2} + \frac{1}{r}\frac{dT_m}{dr} + \left[k^2(sr + t) - \frac{m^2}{r^2}\right]T_m = 0 \quad (3)$$

$$\frac{d^2R_m}{dr^2} + \frac{dR_m}{dr}\left[\frac{1}{r} - \frac{s}{sr + t}\right] + \left[k^2(sr + t) - \frac{m^2}{r^2}\right]R_m = 0 \quad (4)$$

where

$$E_z = \sum_m P_m(\theta)T_m(r) \qquad H_z = \sum_m P_m(\theta)R_m(r)$$

with

$$P_m(\theta) = e^{im\theta}$$

where

$$m = 0, \pm 1, \pm 2, \cdots.$$

The appropriate solutions to these equations, in series form, are indicated in the Appendix. It may be remarked that, while the parallel polarization case (E_z) offers little difficulty, perpendicular polarization (H_z) gives rise to a two-point singularity problem, so that the fields throughout the cylinder cannot be described by a single series solution, for all values of the parameters.

The solution of the diffraction problem proceeds in a well-known manner [2] by expanding the incident wave in cylindrical harmonics, assuming an outgoing (scattered) wave which must have the form of a Hankel function in free space, and matching the tangential components of electric and magnetic field intensities at the surface of the cylinder. This procedure yields for the m^{th} mode reflection coefficient

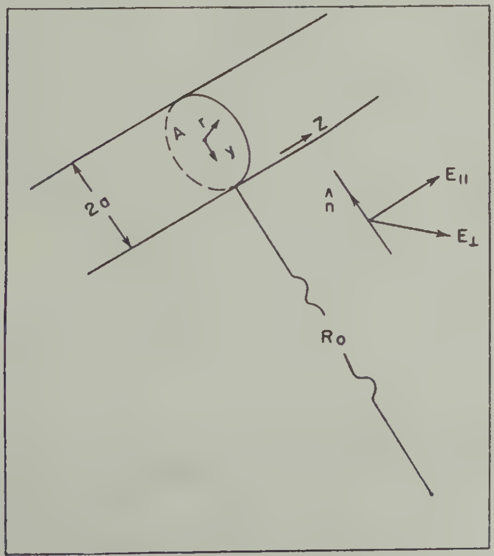
$$r_m = \frac{kT_m(a)J'_m(ka) - J_m(ka)T'_m(a)}{T'_m(a)H_m^{(1)}(ka) - kH_m'^{(1)}(ka)T_m(a)} \quad \text{(5)}$$

and a similar expression involving R_m in the transverse case. The total reflection coefficient, giving the ratio of reflected field intensity at the ground to incident field intensity at the cylinder, normalized through division by $\sqrt{2/\pi k R_0}$ is

$$r = r_0 + 2\sum_{m=1}^{\infty}(-1)^m r_m \quad \text{(6)}$$

While the series for R and T obtained here are convergent for all values of their arguments, they are not useful for purposes of calculation at large values of these arguments because of their slow convergence. Perturbation methods, based on the Bessel-function solutions for a constant density medium may be utilized in this range. As a result of the expansion of the ionized column, and consequent reduction in electron density, however, it is doubtful whether the range of simultaneously large K_0 and ka is ever met in practice. For large ka, and K_0 approaching unity, the wave is not materially reduced in amplitude as it penetrates the cylinder. Each electron is then excited, and radiates, independently of the others. The resultant scattered field may be obtained under these conditions by summing the individual contributions, account being taken of their respective phases. For a given distribution, $N(r)$, this may be written, at normal incidence

$$\text{refl. coeff.} = \frac{\mu_0 e^2}{4\pi m R_0} \iint_A N(r) e^{2iky}\, dA \int_L e^{ikz^2/R^2}\, dz \quad \text{(7)}$$



CYLINDER–WAVE ORIENTATION

In general, a meteor trail is many Fresnel zones in length, so that L may be taken as infinite. The solution for several specific distributions is indicated in the Appendix.

III—*Discussion of theoretical results*

Figure 1 indicates the dependence of the reflection coefficient on the effective average dielectric constant in the region of small ka, for the constant density and linearly varying models. At small values of $ka\sqrt{K_0}$, the reflection coefficients

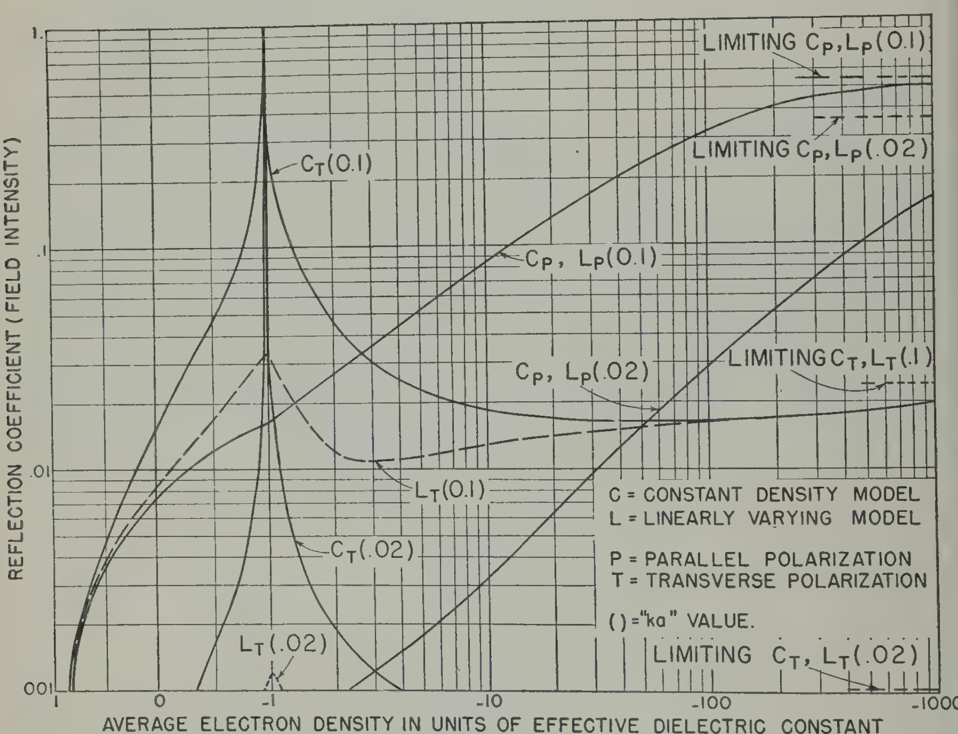


Fig. 1—DEPENDENCE OF REFLECTION COEFFICIENT UPON TRAIL DIAMETER AND ELECTRON DENSITY

for the two polarization directions become independent of the model, and are given by

$$r_\parallel \simeq \frac{\pi}{4}(ka)^2[K_0 - 1] \qquad r_\perp \simeq \frac{\pi}{2}(ka)^2\left[\frac{K_0 - 1}{K_0 + 1}\right]$$

These become equal as K_0 approaches unity; that is, as the electron density falls to low values. As an effective dielectric constant of -1 is approached, the transversely polarized mode exhibits a resonance behaviour, which is much sharper for the constant density than for the linear model. This resonance effect is re-

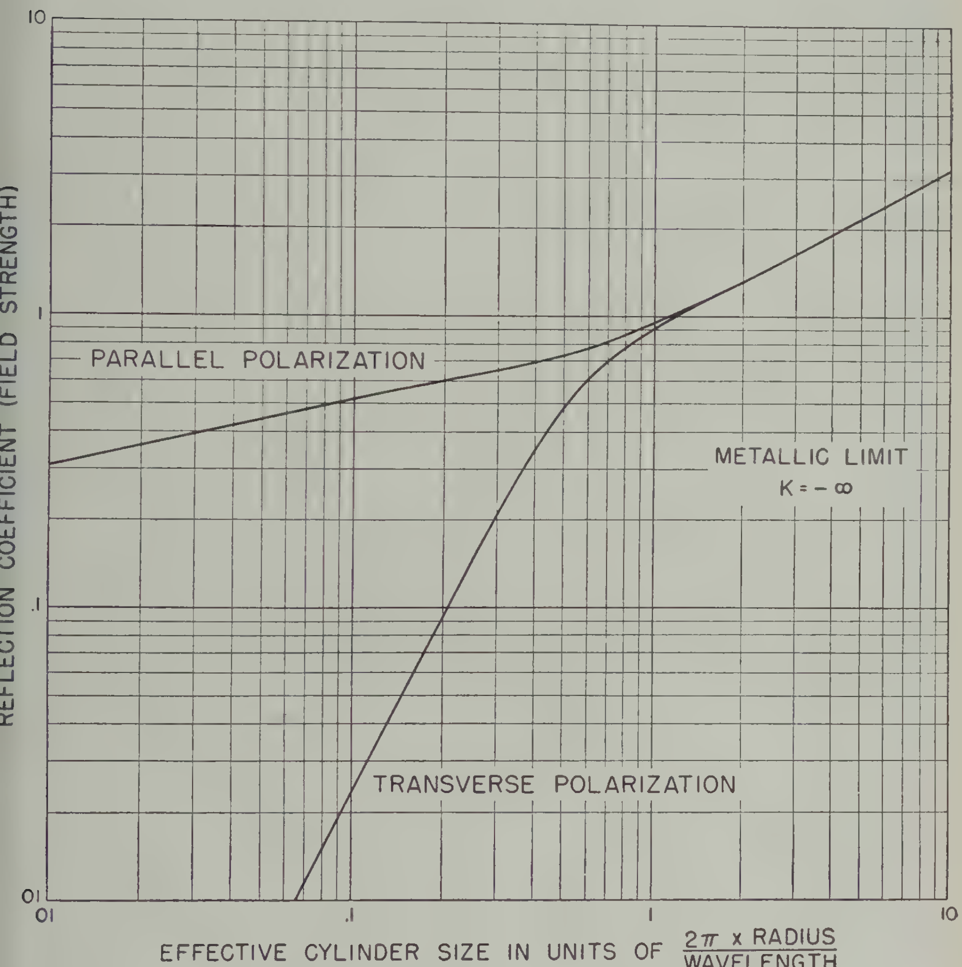


FIG. 2—VARIATION OF LIMITING REFLECTION COEFFICIENT WITH CYLINDER SIZE

markable in that it is independent of ka, although the peak magnitude reached does increase with this parameter. At large values of ka, this resonance effect disappears.

For highly negative dielectrics, metallic type reflection is obtained; the coefficients, for small ka, become

$$r_{\parallel} \simeq \frac{\pi}{2 \ln (\gamma ka/2i)} \qquad r_{\perp} \simeq \frac{3\pi}{4} (ka)^2$$

The reflection coefficient for the parallel mode is much larger than that for the transverse mode in this region, but the rapidity of approach to these limiting values is greater for the latter. In addition, the metallic limit is approached at less negative values of K_0, the larger ka. For a plane surface, the limit of a large cylinder, this limit is reached at $K_0 = 0$.

For large ka, the two mode coefficients become identical. This is indicated in Figures 2 and 3, for several values of dielectric. It should be pointed out, however, that whereas the reflection coefficient for the combination of low electron densities and small cylinders, as well as that for highly negative dielectrics and all size cylinders is essentially independent of the electron distribution function, this ceases to be true for low electron densities contained in large cylinders. This is easily understood in terms of the incident wave penetration through such cylinders, and the resultant phase differences between the oscillating electrons. The large differences in behaviour to be expected are indicated by Figure 4, wherein the dependence of the reflection coefficient upon ka, normalized with respect to total number of electrons, is given for the constant density, linear, and Gaussian-type distributions. The latter is a highly damped affair, while the other two are oscillating functions. The general statement may be made that the sharper the boundary associated with a distribution of charge, the more slowly does the envelope of the signal fall with increase in cylinder size. For the linear case, for example, this rate is proportional to $1/(ka)^{5/2}$, while for the constant density model it is $1/(ka)^{3/2}$, and for a shell of thickness small compared to a wavelength, $1/(ka)^{1/2}$.

The following Table summarizes the gross features of the various combinations.

Trail parameters		Effect on reflection coefficient of	
Cylinder size	Average electron density	Polarization	Distribution
Small	Low	Small	Small
	High	Large	Small
Large	Low	Small	Large
	High	Small	Small

From the experimental viewpoint, this outline is suggestive of the procedures to be followed in obtaining values for trail parameters at various stages during the decay process. Polarization measurements should prove useful immediately after trail formation, but will cease to be fruitful rather rapidly. Multifrequency determinations may be utilized to pass from one region of the above Table to an-

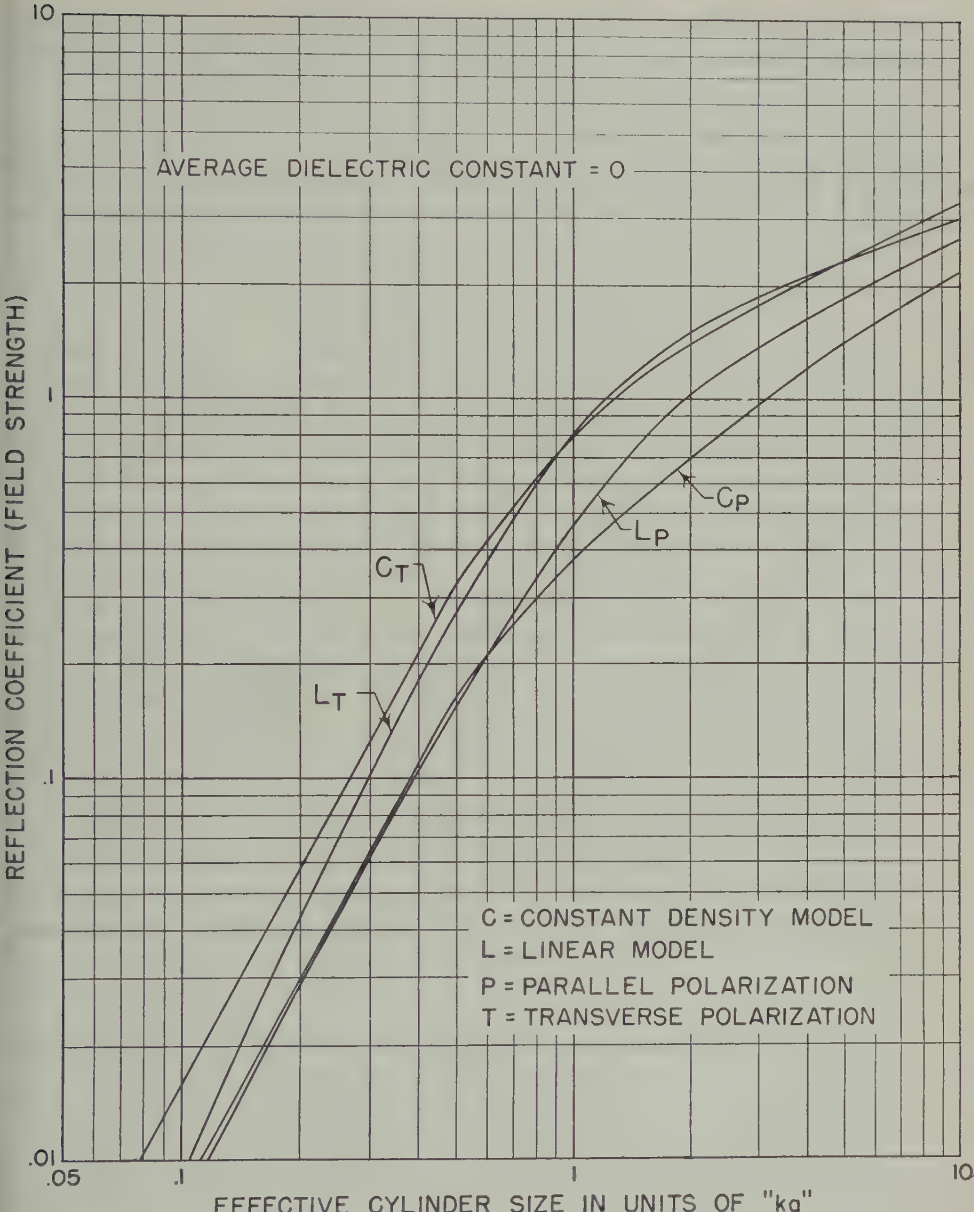


FIG. 3—TYPICAL BEHAVIOUR IN INTERMEDIATE RANGE OF PARAMETERS

other. The behaviour in the middle range of cylinder size and dielectrics lies in the region of transition between the above categories. The general nature of these transitions is illustrated in the various Figures.

The above results have been obtained on the assumption of normal incidence of the transmitted wave upon a smooth cylinder. If the geometry of the system is such as to make this impossible, one must examine the type of behaviour to be expected. For a perfectly smooth cylinder, it may be shown that the ratio of the off-perpendicular signal to that obtained at normal incidence is $1/2\pi \sin \theta_0 \sqrt{R_0/\lambda}$, where R_0 is the observation site-trail distance, and θ_0 is the minimum angle of incidence. This holds for $\theta_0 > 0°.1$, for the usual values of the remaining param-

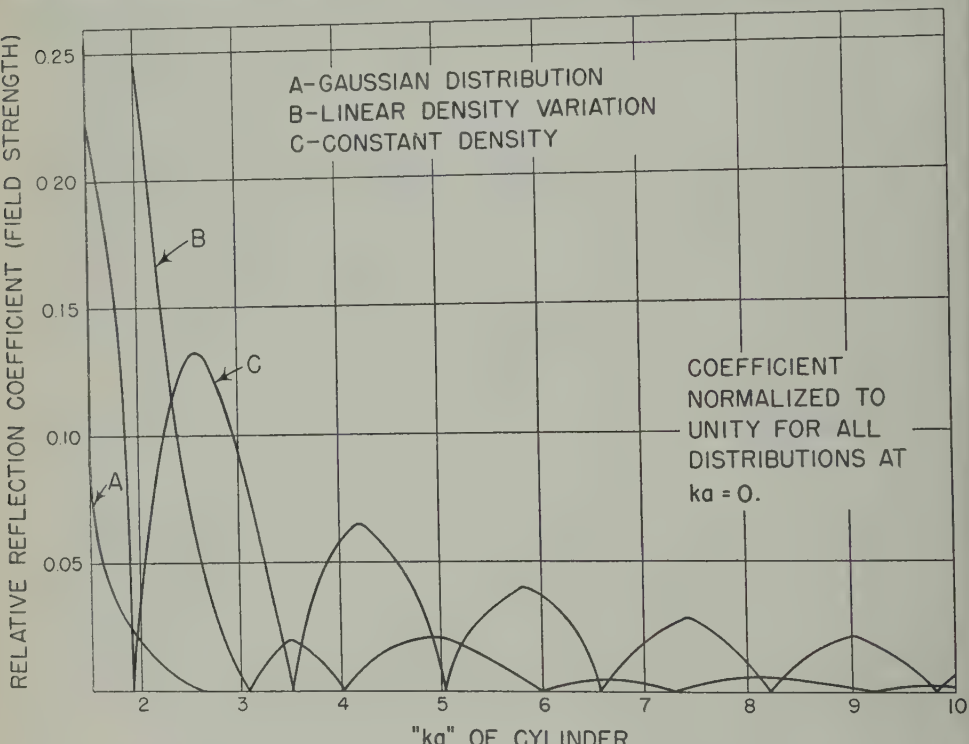


FIG. 4—DEPENDENCE OF REFLECTION COEFFICIENT ON ELECTRON DISTRIBUTION FOR LARGE CYLINDERS OF LOW ELECTRON DENSITY

eters; for nominal values of θ_0, the ratio is of the order 10^{-3}. If the cylinder of ionization is not smooth, as a result of turbulence, two cases must be distinguished, in accordance with the value of electron density. For high electron density, the reflection is essentially a surface affair. In this event, the magnitude of the off-perpendicular reflection depends upon the scale of the surface roughness. If the latter is small (compared to a wavelength), the cylinder will behave as though it were smooth; while if it is large, the reflection will be diffuse and the energy returned may begin to approach that of the normal incidence case. It should be pointed out that if the roughness possesses regular features, a diffraction pattern will result which can give rise to highly singular effects at a single observation

point. For low electron densities, the reflection is a volume affair, so that it is turbulence of this latter type which enters. Considerations similar to those discussed above apply to the scale of this roughness, but, in addition, the mean departure of the electron density from uniformity enters in determining the returned signal amplitude. The scattering theory recently given by Booker and Gordon [3] may be employed in this case. It predicts a scattered signal in the backward direction proportional to λ^2, provided the scale of the turbulence is of the order of, or greater than, the wavelength.

IV—*Some applications to experimental results*

The interpretation of the wide variety of echo patterns obtained at the many laboratories employing radar techniques in the study of meteors has been based mainly on recombination and diffusion as the primary mechanisms responsible for the trail decay. This has given rise to some puzzling questions, and produced some rather curious statements in the literature. For example, the ratio of almost 1,000:1 in the time duration of different echoes requires for an explanation based on the above mechanism an anisotropy in the diffusion coefficient of corresponding magnitude [4], since recombination would effectively reduce all high-density electron concentrations rather quickly, compared to their dispersal through normal diffusive expansion. But the ratio of diffusion transverse to the earth's magnetic field, to that parallel to it, is $1{:}1 + (w\tau)^2$ where w = gyro-frequency for the dominant constituent; τ = mean collision time [5].

For the positive ions, the controlling constituent in determining the diffusion rate [6], $w\tau$, is less than 0.1, so that a difference of the order of one per cent only is to be expected from this source.

Experimentally, deep fluctuations in amplitude of remarkably constant average period are obtained in many echoes [7], as contrasted with the highly damped signal predicted by the exponential distribution characteristic of normal diffusion. Rather special type winds have been invoked to explain data of this sort, but the depth and the regularity of the fading make such a mechanism improbable in most cases.

It is the purpose of this section to present a qualitative picture of the decay period by interpreting the experimental results reported in the literature in the light of section III. A paper is in course of preparation which will give numerical results based on the observations made by the National Bureau of Standards.

Inspection of Figure 4 indicates that, while the exponential distribution produced by normal diffusion does not give rise to a fluctuating reflection coefficient, one containing a sharper boundary does result in such a pattern. In addition, the rather low rate of fall with time of the envelope of these fluctuations, obtained experimentally, indicates a fairly abrupt discontinuity in the distribution function. As to the mechanism giving rise to such a structure, it is the opinion of the author that a good case can be made for a shock-wave type expansion. The high temperature which must exist during the trail formation, coupled with mean-free-path considerations at 100 km, constitutes a good setting for the creation of such a wave; in addition, the rapid trail expansion called for by the experimentally-determined fading period could be met by such a mechanism. Since such a wave

would be formed within a few collision times of the high energy particles in the trail, the almost immediate reduction in charge density would prevent recombination from playing a major role, thereby maintaining initial electron content through the decay period. Differences in meteor size are then sufficient to explain the observed range of echo lifetimes. In addition, the electrons produced by auto-ionization of metastable states, for which there is some theoretical evidence, would be subject to a much lower recombination rate. The lack of amplitude fluctuations on some echoes can be ascribed to the existence of a critical meteor size, below which the energy produced is not sufficient to give rise to a shock wave. A detailed examination of echo decay rates should provide additional information on this point. It should be pointed out that a cylindrical shock wave will ultimately dissipate its energy, so that the final stage of the decay process, even for large meteors, is probably governed by turbulence and diffusion. Since these processes act rather slowly, it is to be expected that the fading period would lengthen once the shock wave subsided. There appears to be experimental evidence verifying this point [7].

A possible explanation for the echo stratification effects reported by McKinley and Millman [8] may be advanced on the basis of the theory of off-perpendicular reflection outlined in the last section. It is known that the direction and magnitude of winds and associated turbulence vary rapidly from one level to another in the upper atmosphere. A meteor trail which is formed intersecting two or more levels of turbulence but in such a position that radio pulses are off perpendicular, might nevertheless return detectable echoes from those portions of the trail lying in the turbulent zones.

While the number of independent factors present in any geophysical phenomenon is such as to preclude the applicability of a single explanation or model to all observations, the picture presented above, with suitable modifications drawn from the theory of the preceding sections to clarify more detailed experimental results, does appear to be consistent with the bulk of observational data.

In conclusion, I should like to express my thanks to Mr. T. N. Gautier, of this Laboratory, for many illuminating discussions.

Appendix

For a time dependence of the form exp $(-iwt)$, Maxwell's equations become

$$\nabla \times \mathbf{E} = iw\mathbf{B} \qquad (1a)$$

$$\nabla \times \mathbf{H} = -iw\epsilon_0\mathbf{E} + \mathbf{J} \qquad (2a)$$

The equation of motion for a single electron:

$$m\dot{\mathbf{V}} = e\mathbf{E} - m\nu\mathbf{V} \qquad (3a)$$

$$\mathbf{V} = \frac{ie\mathbf{E}}{umw} \qquad (4a)$$

where $u = 1 + i(\nu/w)$ and ν electron collision frequency

$$\mathbf{J} = Ne\mathbf{V} = \frac{iNe^2\mathbf{E}}{mwu} \quad \text{......(5a)}$$

Substituting in (2a)

$$\nabla \times \mathbf{H} = -iw\epsilon_0\left[1 - \frac{Ne^2}{\epsilon_0 mw^2 u}\right]\mathbf{E} \quad \text{......(2b)}$$

$$= -iw\epsilon_0 KE$$

where $K = 1 - \dfrac{Ne^2}{\epsilon_0 mw^2 u}$, the effective dielectric constant.

$$\nabla \times (\nabla \times \mathbf{E}) = iw\mu_0\epsilon_0 K(-iw)\mathbf{E} \quad \text{......(1b)}$$

$$\nabla^2\mathbf{E} - \nabla(\nabla\cdot\mathbf{E}) + \frac{w^2}{c^2}K\mathbf{E} = 0 \quad \text{......(1)}$$

$$\nabla \times (\nabla \times \mathbf{H}) = -iw\epsilon_0[K(\nabla \times \mathbf{E}) - \mathbf{E} \times \nabla K] \quad \text{......(2c)}$$

$$\nabla^2 H + \frac{w^2}{c^2}K\mathbf{H} - (\nabla \times \mathbf{H}) \times \frac{\nabla K}{K} = 0 \quad \text{......(2)}$$

After separating variables, through the substitution

$$E_z = P(\theta)T(r)$$

one obtains

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \left[k^2(sr + t) - \frac{m^2}{r^2}\right]T = 0 \quad \text{......(3)}$$

Assume power-series solution

$$T = r^n \sum_{q=0}^{\infty} c_q r^q$$

indicial equation

$$N(N - 1) + N - m^2 = 0$$

$$N = \pm m$$

where the + sign must be employed for the solution to be valid at the origin.

Recurrence relation:

$$c_q = -\frac{k^2t}{(m + q)^2 - m^2}c_{q-2} - \frac{k^2s}{(m + q)^2 - m^2}c_{q-3}$$

$$T_m = r^m\left[1 - \frac{k^2t}{(m + 2)^2 - m^2}r^2 - \frac{k^2s}{(m + 3)^2 - m^2}r^3 + \frac{(k^2t)^2}{[(m + 4)^2 - m^2][(m + 2)^2 - m^2]}r^4 + \cdots\right] \quad \text{......(3b)}$$

To obtain the relation between the parameters of the distribution and the average dielectric, we define

$$K_0 = \frac{1}{\pi a^2}\int_0^a 2\pi r K(r)\,\mathrm{d}r$$

$$= 1 - \frac{as}{3}$$

where $K(r) = sr + t$ and $K(a) = 1$, so that $t = 1 - as$.

For the H_z (transverse polarization) case

$$\nabla^2 H_z + k^2 K H_z - \frac{1}{K}\cdot\frac{\mathrm{d}K}{\mathrm{d}r}\cdot\frac{\partial H_z}{\partial r} = 0 \qquad (2d)$$

After separating variables through the substitution $H_z = P(\theta)R(r)$, one obtains

$$\frac{\mathrm{d}^2 R}{\mathrm{d}r^2} + \frac{\mathrm{d}R}{\mathrm{d}r}\left[\frac{1}{r} - \frac{s}{sr+t}\right] + \left[k^2(sr+t) - \frac{m^2}{r^2}\right]R = 0 \qquad (4)$$

It is desirable to expand about the singularity $rs + t = 0$. Therefore, let $x = r - u$, $u = -t/s$.

$$x(x+u)^2\frac{\mathrm{d}^2R}{\mathrm{d}x^2} - u(x+u)\frac{\mathrm{d}R}{\mathrm{d}x} + [k^2 s x^2 (x+u)^2 - m^2 x]R = 0 \qquad (4b)$$

$$R = x^n \sum_{q=0}^{\infty} c_q x^q$$

The indicial equation $u^2 n\,(n-1) - u^2 n = 0$, yields $n = 0, 2$.

$$\left.\begin{aligned} c_q = &-\frac{1}{u^2(n+q)(n+q-2)}\{u(q+n-1)(2q+2n-5)c_{q-1} \\ &+ [(n+q-2)(n+q-3) - m^2]c_{q-2} \\ &+ (k^2su^2)c_{q-3} + (2k^2su)c_{q-4} + (k^2s)c_{q-5}\} \\ R = x^n &\left\{1 - \frac{n(2n-3)}{(n+1)(n-1)}\left(\frac{x}{u}\right)\right. \\ &\left.+ \left[\frac{(2n-1)(2n-3)}{(n+2)(n-1)} - \frac{n(n-1)-m^2}{n(n+2)}\right]\left(\frac{x}{u}\right)^2 + \cdots\right\} \end{aligned}\right\} \qquad (4c)$$

As a result of the degeneracy of the indicial equation, it is simplest to take as independent solutions the pair

$$R_A = nR\Big]_{n=0} \qquad \text{and} \qquad \frac{\mathrm{d}(nR)}{\mathrm{d}n}\Big]_{n=0} = R_A \log(kx) + R_B$$

where

$$R_A = \frac{m^2}{2}\left(\frac{x}{u}\right)^2 - \frac{m^2}{2}\left(\frac{2\cdot 1}{3\cdot 1}\right)\left(\frac{x}{u}\right)^3 + \frac{m^2}{2}\left[\left(\frac{2\cdot 1}{3\cdot 1}\right)\left(\frac{3\cdot 3}{4\cdot 2}\right) - \frac{2\cdot 1 - m^2}{4\cdot 2}\right]\left(\frac{x}{u}\right)^4 + \cdots$$

$$R_B = 1 - \left[1 + \frac{m^2}{4}\right]\left(\frac{x}{u}\right)^2 + \left[\frac{2 - (ku)^2 su - \frac{2}{3}m^2}{3\cdot 1}\right]\left(\frac{x}{u}\right)^3 + \cdots \quad (4d)$$

At $r = 0$, $x = u$, both these solutions diverge as harmonic series.

To maintain the solution finite at this point, it is consequently necessary to obtain the limiting coefficients of the divergent portions of each of these series at $r = 0$, utilizing these to combine the independent solutions so that the infinite portions cancel.

In general, these limiting numbers must be found individually for each set of parameters, and for every mode. For small $K_0(ka)^2$, however, the determination of the ratios in which the independent solutions combine becomes essentially independent of the parameters. For

$$R = AR_A + B(R_A \log kx + R_B) \quad (4e)$$

one obtains for the dominant mode

$$\left(\frac{A}{B}\right)_{m=1} = -i\pi - \log(ku) + 1.21$$

The domain of convergence of the above solution, modified in this manner, is

$$\left|\frac{r}{u} - 1\right| \leq 1,$$

or everywhere excluding the region $1/3 < K_0 < 5/6$. Within this last range of K_0, the solution must be expanded about $r = 0$; for $K_0 > 2/3$, no other singularity exists, that is, the functional dielectric never reaches zero; for $K_0 < 2/3$ (and $>1/3$), a matching of infinities at the zero dielectric radius enables the desired solution to be obtained. For $K_0 = 2/3$, the dielectric becomes zero at the origin, so that the two singularities merge. In this case, the differential equation reduces to the form

$$\frac{d^2R}{dr^2} + \left[k^2\frac{r}{a} - \frac{m^2}{r^2}\right]R = 0 \quad (4f)$$

which yields as indices

$$n = \frac{1 \pm \sqrt{1 + 4m^2}}{2}$$

the positive sign being taken before the radical for the solution to remain finite at $r = 0$, for $m \neq 0$. For the $m = 0$ mode, we must take the $n = 0$ solution to maintain the electric field, $1/K\, dR/dr$, finite at $r = 0$. The recurrence relation is quite simple, as follows:

$$c_q = -c_{q-3}\left[\frac{k^2/a}{(n + q + 2)(n + q + 1) - n(n - 1)}\right]$$

To derive the expression for the reflection coefficient, we take, in the transverse case

$$\left.\begin{aligned} r < a: \quad & H_z = \sum_{m=-\infty}^{\infty} c_m R_m(r) e^{im\theta} \\ r > a: \quad & H_z = \sum_{m=-\infty}^{\infty} H_0 i^m J_m(kr) e^{im\theta} + d_m H_m^{(1)}(kr) e^{im\theta} \end{aligned}\right\} \qquad (5b)$$

the sum of the incident and outward scattered waves. The electric fields are then found from relation (2b)

$$\left.\begin{aligned} r < a: \quad & E_\theta = \frac{1}{\epsilon_0 i\omega K} \sum_{m=-\infty}^{\infty} c_m R_m'(r) e^{im\theta} \\ r > a: \quad & E_\theta = \frac{k}{\epsilon_0 i\omega} \sum_{m=-\infty}^{\infty} [H_0 i^m J_m'(kr) + d_m H_m'^{(1)}(kr)] e^{im\theta} \end{aligned}\right\} \qquad (5c)$$

Equating these expressions at $r = a$, $K(r) = 1$, yields equation (5) of the text.

To evaluate equation (7) for a constant density distribution: $N(r) = n_0$

$$\left.\begin{aligned} \iint_A &= n_0 \int_0^a \int_0^{2\pi} \cos(2kr\sin\theta) r \, d\theta \, dr \\ &= n_0 \int_0^a 2\pi r J_0(2kr) \, dr \\ &= N_0 \left[\frac{J_1(2ka)}{ka}\right] \end{aligned}\right\} \qquad (7a)$$

where $N_0 = \pi a^2 n_0$

For a linear distribution: $N(r) = n_0(1 - r/a)$

$$\begin{aligned} \iint_A &= \int_0^a \int_0^{2\pi} r\left(1 - \frac{r}{a}\right) \cos(2kr\sin\theta) \, d\theta \, dr \\ &= 2\pi n_0 \left[\int_0^a r J_0(2kr) \, dr - \frac{1}{a} \int_0^a r^2 J_0(2kr) \, dr\right] \\ &= N_0 \left[-\frac{3}{2(ka)^2} J_0(2ka) + \frac{3}{4(ka)^3} \int_0^{2ka} J_0(x) \, dx\right] \end{aligned}$$

where $N_0 = \pi/3 \; n_0 a^2$, again the total number of electrons per unit length. The last integral is tabulated directly for some values of the upper limit, but is better expressed in terms of Struve functions, which are given at .02 argument interval by Watson [9].

$$\iint_A = \frac{N_0}{(ka)^2}\left[\frac{3\pi}{4}\{J_1(2ka)\mathbf{H}_0(2ka) - J_0(2ka)\mathbf{H}_1(2ka)\}\right] \qquad (7b)$$

where $\mathbf{H}_0$, $\mathbf{H}_1$ are the Struve functions of the zeroth and first order, respectively.

For a Gaussian distribution: $N(r) = n_0 e^{-(r/a)^2}$

$$\iint_A = n_0 \int_0^\infty \int_0^{2\pi} e^{-r^2/a^2} \cos(2kr \sin\theta) r \, d\theta \, dr = N_0 e^{-(ka)^2} \Big\} \qquad (7c)$$

where $N_0 = \pi n_0 a^2$

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FINE STRUCTURE OF THE LOWER IONOSPHERE*

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ABSTRACT

Pulse soundings of the ionosphere at vertical incidence taken at 100 and 325 kc show pronounced splitting of the echoes received at night. The ranges of most of the echo components are found to lie between about 90 and 130 km. Four possible explanations of splitting are suggested, as follows: (1) Off-vertical reflections, (2) magneto-ionic splitting, (3) partial reflection in a layer of monotonically increasing ionization-density, and (4) division of the region into a series of partially-reflecting partially-transmitting strata. Experimental data in the form of records of virtual height as a function of time indicate that the first three explanations are not acceptable. The fourth explanation, however, is consistent with all available data, including the characteristics of multiple reflections and certain systematic differences in the polarizations of echoes at different heights. It is concluded, therefore, that the lower ionosphere contains at night a system of reflecting strata at discrete heights, capable of both reflecting and transmitting incident electromagnetic energy. A new mechanism of ionospheric layer formation may be required to explain these results.

I—INTRODUCTION

Recent widespread interest in the physical and electrical properties of the lower regions of the ionosphere, from both the upper-atmosphere physics and wave-propagation points of view, has stimulated the performance of experiments aimed at extending our knowledge of this important part of the earth's atmosphere. Although much is known about the normal daytime E-layer, detailed knowledge of the D-region and the night-time E-region is lacking. Data on the virtual height and reflection coefficient of these regions have been obtained through the use of continuous-wave methods and the study of atmospheric wave-forms, but the inherent limitations of these techniques have left unanswered many questions regarding their physical and electrical properties.

Extension of our knowledge of the lower ionosphere is now possible with the development of a vertical-incidence ionospheric pulse-sounding technique for use

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at low frequencies [see 1 of "References" at end of paper]. It is the purpose of this paper to describe the results of the application of this technique at frequencies of 100 and 325 kc, and to show that they demonstrate the existence at night of a multiplicity of ionized strata lying in the height range of about 90-130 km. The discussion will be confined to those observations which seem to be characteristic of the undisturbed ionosphere.

II—PREVIOUS WORK

The possibility of a fine structure in the lower ionosphere is suggested in the works of a number of experimenters whose observations are cited below.

A fine-structure in the upper part of the *E*-region was first reported by E. V. Appleton [2]. This was observed most frequently in early morning hours and has been known as the *E*2-layer.

In the year 1933, Ratcliffe and White [3], during the course of polarization experiments using pulses, noted the occasional doubling of night-time abnormal-*E* echoes. They found that the component separation did not increase with order of reflection as in the case of magneto-ionic splitting in the *F*2-layer, and that both components had the same polarization at any instant. They concluded, therefore, that this doubling phenomenon was due to double stratification in the region.

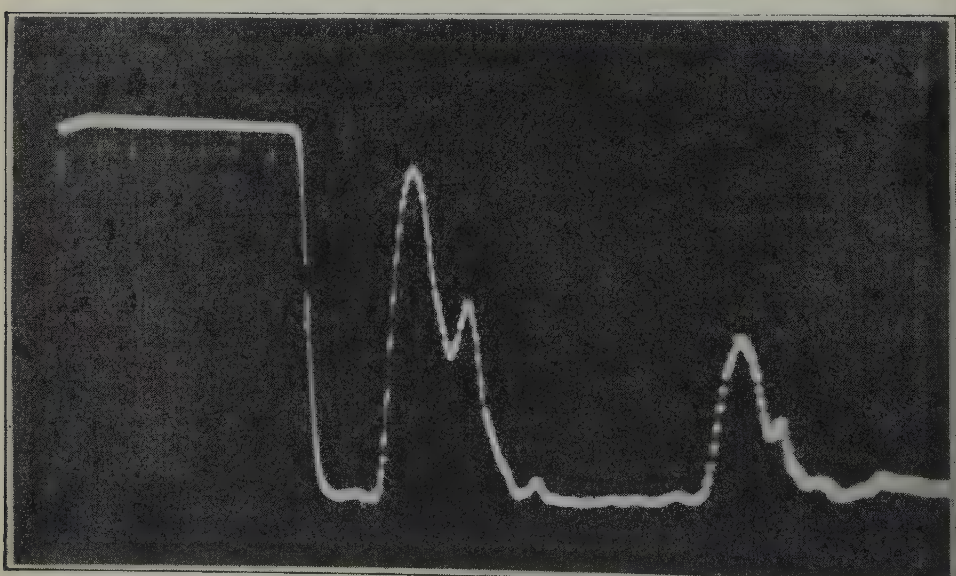

Fig. 1—*A*-scan record, 100 kc, 22 March 1948, $21^h\,55^m$ PST, showing split *E*-reflection at 90 km and first multiple at 180 km. Note overlapping of pulse components resulting from excessive pulse-length. Time constant of exponential pulse = 150μ sec; receiver band-width = 25 kc; 10-km height markers.

Evidence of stratification below the normal *E*-layer was obtained by Smith and Kirby [4] in 1937 from field-intensity records of a broadcast station, having a frequency of 1,040 kc, and path length of 480 km. In the early morning hours,

they observed an abrupt increase in field intensity which they attributed to a shift of the reflection point from the E-layer to one at a lower height.

Appleton, Naismith, and Ingram [5] observed weak echoes in the 90-100 km range, well below normal E, throughout the whole of the high-frequency range at times when the region was otherwise normal.

E-region stratification evidence of a different nature was obtained recently by McKinley and Millman [6] who found that the radar traces of long enduring echoes from meteoric ionization were often split with range separations averaging 3-4 km. However, the extent and orientation of the ionization structure could not be determined from their data.

The first vertical-incidence observations using pulses at low frequencies showed that lengthening and distortion of the trailing part of the echoes were dominant phenomena [7], during night-time at least. An example of these early results is shown in Figure 1. The characteristics of the pips in the tails of the echoes were such as to suggest, but by no means prove, the presence of a system of partially transparent horizontal layers in the E-region. With this hypothesis in mind, the experimental technique was appropriately modified and further soundings were made in an attempt to discover the cause of splitting. The results of these experiments are described and analyzed below.

III—ANALYSIS OF HIGH-RESOLUTION LOW-FREQUENCY PULSE SOUNDINGS

The sounding technique was modified to provide greater resolution between closely-spaced reflected pulses. The effect of reduced pulse-length and increased receiver band-width is shown in Figure 2, in which it can be seen that the separation

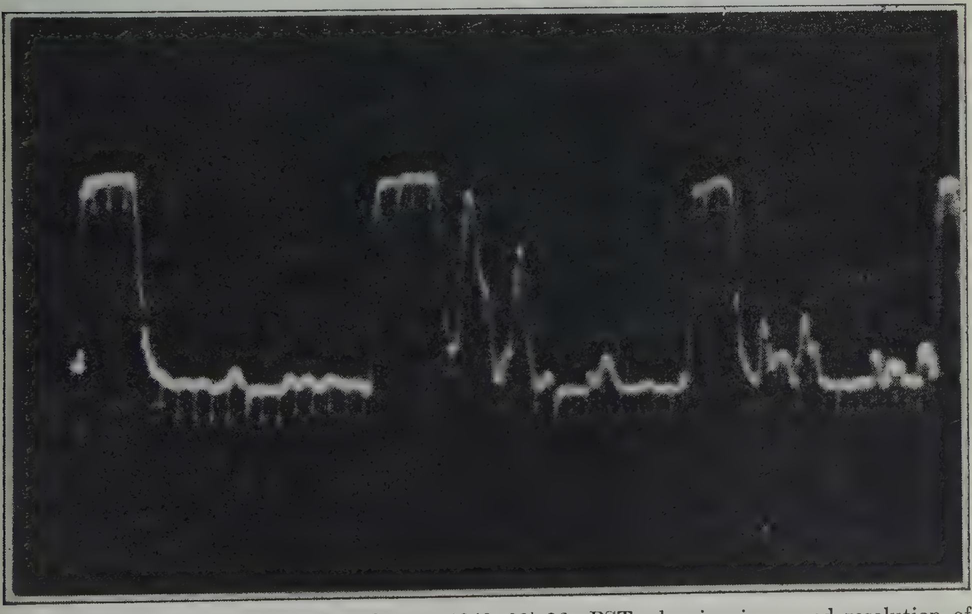

Fig. 2—A-scan record, 100 kc, 21 February 1949, $00^h\ 06^m$ PST, showing improved resolution of split E-region echoes resulting from reduced pulse-time constant (30μ sec) and increased receiver band-width (50 kc); 5-km height markers.

of the components in the split reflection is almost complete. Trace definition was further improved by differentiating the signals leaving the detector, giving an effective recorded pulse-length of 5 km or less. The A-scan trace was replaced by an intensity-modulated trace recorded on slowly-moving photographic film. Original records illustrating the results obtained with the improved technique are shown in Figures 3A, 4A, 5A, and 6. A characteristic feature of such records as these is the multiplicity of echoes in the height region of 90-130 km. The observed heights of the components of split echoes, while often constant over periods of several hours, are found to vary widely from day to day.

In looking for an explanation of echo splitting, the following possibilities are considered:

(1) Off-vertical reflections
(2) Magneto-ionic splitting
(3) Partial reflection in a layer of monotonically increasing ionization-density, at levels of anomalous type variation of refractive index
(4) Division of the region into a series of partially-reflecting partially-transmitting strata

The first explanation, in terms of off-vertical reflections, is not consistent with the main features of the data. Consider Figures 3A and B, for example. Two traces

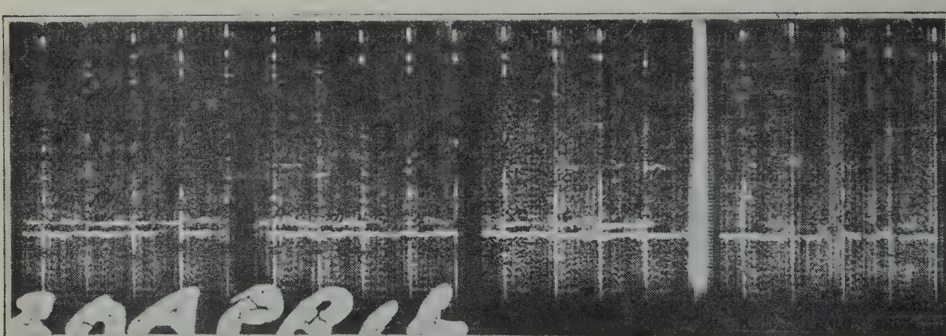

FIG. 3A—Original $h't$ record, 325 kc, 20 April 1949, $22^h\ 05^m$-$22^h\ 25^m$ PST, showing doubling of E-echoes. Pulse-time constant $< 30\mu$ sec; receiver band-width = 50 kc; 5-km height markers appear at five-minute intervals. Reception alternated between crossed N-S and E-W receiving dipoles every 30 seconds; N-S recording period adjacent to height-marker interval.

are observed at heights of about 100 and 120 km. Although the intensities of each trace vary markedly in the two-hour period shown, there is relatively little change in range on either trace. For the upper trace to be the result of reflection from a region not overhead would require, of course, a very special layer-geometry which is fixed over appreciable periods of time with respect to the observing station. This appears to be a most unlikely situation from the physical point of view. An independent check for the existence of off-vertical reflections was made by recording with a vertical antenna in place of the horizontal dipole. An example of the results is shown in Figure 4A, where the blank spaces correspond to the

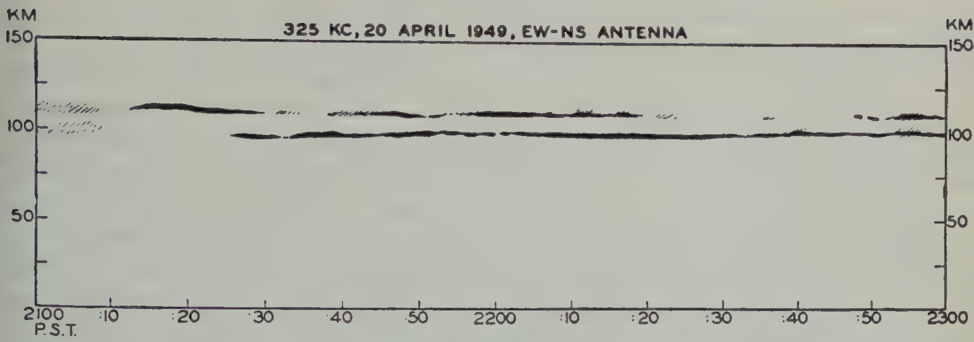

FIG. 3B—Drawing of $h't$ record, a portion of which is reproduced in Fig. 3A, in which polarization effects have been smoothed out. This illustrates the relatively independent intensity variations of the two prominent E-region traces which appeared at heights of 100 and 120 km.

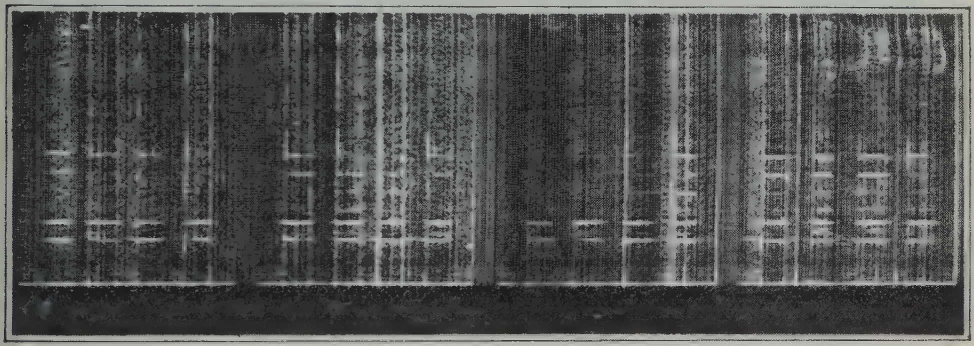

FIG. 4A—Original $h't$ record, 325 kc, 14 June 1949, 02^h10^m-02^h 30^m PST, showing relation of multiple structure to split echoes. Blank spaces at one-minute intervals correspond to recording using a vertical antenna. Reflections received on an east-west dipole.

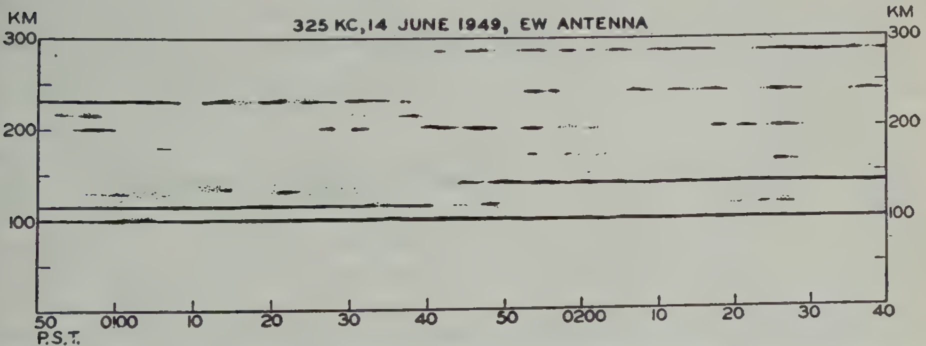

FIG. 4B—Drawing of $h't$ record, a portion of which is reproduced in Fig. 4A, in which traces were interpolated in the blank spaces. Note combination heights present in multiple structure. Note stability of trace heights during periods of fading.

vertical antenna, the sensitivity of which was such that signals arriving at angles of more than five degrees from the vertical would have been observed. The conclusion, therefore, is that the large majority of split echoes are returned from sub-

stantially overhead, and that the phenomenon of splitting is not due to off-vertical reflections.

A second explanation for the observed echo splitting is magneto-ionic separation. This presents a more difficult problem in analysis, since the experimental data suggest strongly that the ionization gradients encountered at night at low frequencies are sufficiently high to invalidate the assumption of "slowly-varying" conditions upon which ordinary ray-theory calculations are based. It is not possible, therefore, to predict with any certainty the nature of propagation at these frequencies, using currently available ray-theory. It is found experimentally, however, that often there is a systematic difference in the polarizations of the components of split echoes, which is suggestive of magneto-ionic splitting. An example of this effect is shown in Figures 5*A* and *B*, in which two split components can be

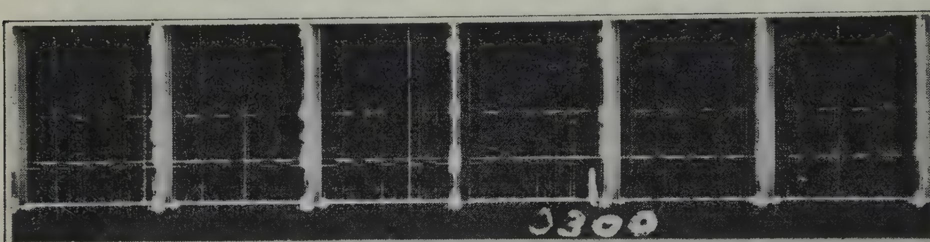

Fig. 5*A*—Original *h't* record, 100 kc, 14 May 1949, $02^h\ 40^m$-$03^h\ 10^m$ PST, showing the effect on height of orientation of receiving antenna. Reception alternated every 30 seconds between two crossed horizontal dipoles, oriented north-south and east-west. North-south periods lie adjacent to the height-marker breaks.

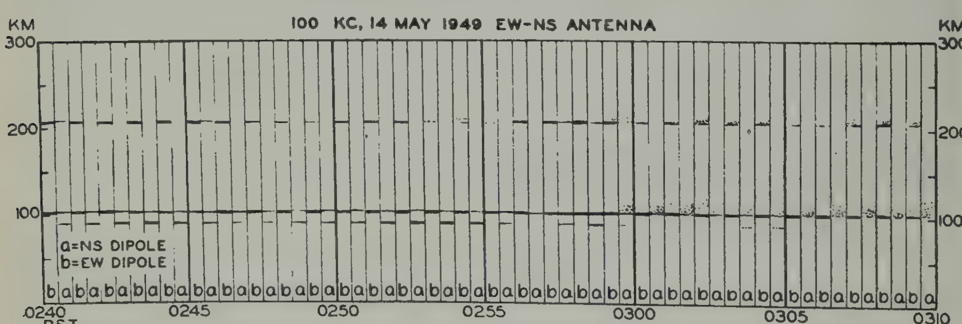

Fig. 5*B*—Drawing of above record. Lower trace at 90 km is observed only on the north-south dipole, while the upper trace at 104 km appears predominantly on the east-west dipole.

observed at heights of about 90 and 104 km. Reception alternated every 30 seconds between two mutually-perpendicular horizontal dipoles, one directed north-south* and the other east-west. The lower component of the split echo was observed only when the north-south dipole was used, while the upper component was observed on both dipoles but with appreciably greater strength on the east-west dipole. The first multiple of the upper component was also stronger on the east-west

*Actually the north-south dipole was about nine degrees west of magnetic north.

dipole. Although other polarization effects were noted, that described above represents the dominant characteristic of split reflections observed at both 100 and 325 kc during the spring and summer of 1949.

A complete discussion of the observed polarization phenomena is not considered to be within the scope of this paper and will be dealt with at a later time. Although an analysis of the full significance of the polarization results is not given, an important inference, nevertheless, may be drawn from the above results. The presence of split reflections of characteristically different polarization implies strongly the existence of two characteristic waves resulting from the action of the earth's magnetic field. This conclusion is at variance with the theoretical calculations of Benner, Grace, and Kelso [8], who found that using ray theory there would be but one magneto-ionic component reflected at 150 kc at the location of State College, Pennsylvania. Available evidence indicates that the assumptions they used would not be altered significantly by the difference in locations. These authors also find in their preliminary results no evidence of systematic difference in the polarizations of the components of split echoes and advance this as evidence that splitting is not magneto-ionic. Since pronounced differences have been found in the Stanford results, it would appear that this argument cannot generally be used to show that splitting is not magneto-ionic. Although there is evidence that both magneto-ionic modes must be considered, other features of the data indicate that the observed splitting is not primarily the result of separation on this basis. If the observed splitting is to be explained by magneto-ionic separation in a layer of monotonically increasing ion-density, then one must assume relatively small ion-gradients because of the large height separations often observed in the split components. It is found experimentally, however (for example, by comparison of simultaneous records on widely separated frequencies), that the apparent gradients are relatively high and not consistent with those required for the magneto-ionic explanation. Furthermore, for small gradients of ionization, one would expect little coupling between the magneto-ionic components, with the result that the separation of split components should increase directly with the order of reflection as it normally does at high frequencies. Virtually no evidence of this has been found at either 100 or 325 kc; on the contrary, the separation is independent of order.

A third explanation of splitting is partial reflection in a layer of monotonically increasing ionization-density. Under certain conditions the refractive index for a single layer can change abruptly at levels other than those at which it is reduced to zero. This anomalous type of refractive-index variation could conceivably give rise to partial reflection. It is instructive to examine the characteristics of splitting with this possibility in mind. Consider the record shown in Figures 4*A* and *B*, for example. From $00^h\,00^m$ to about $01^h\,40^m$ PST there can be seen two principal first-order reflections, the first at about 100 km and the second 15 km above it. At about $01^h\,40^m$ the upper component fades out, while at the same time a new trace, at about 140 km, appears. Later, at about $02^h\,00^m$, reflections at the intermediate height reappear (with negligible change in height) for a short time. The significant point to note is that, although there are marked changes in the intensities of the various split components with time, there are no corresponding

changes in height. To explain how the properties of a single layer might be altered in such a way as to affect so markedly the intensities of the split components without affecting their virtual heights is most difficult. The small changes of reflection heights with respect to large changes in frequencies (see Fig. 6) cast further doubt on this hypothesis.

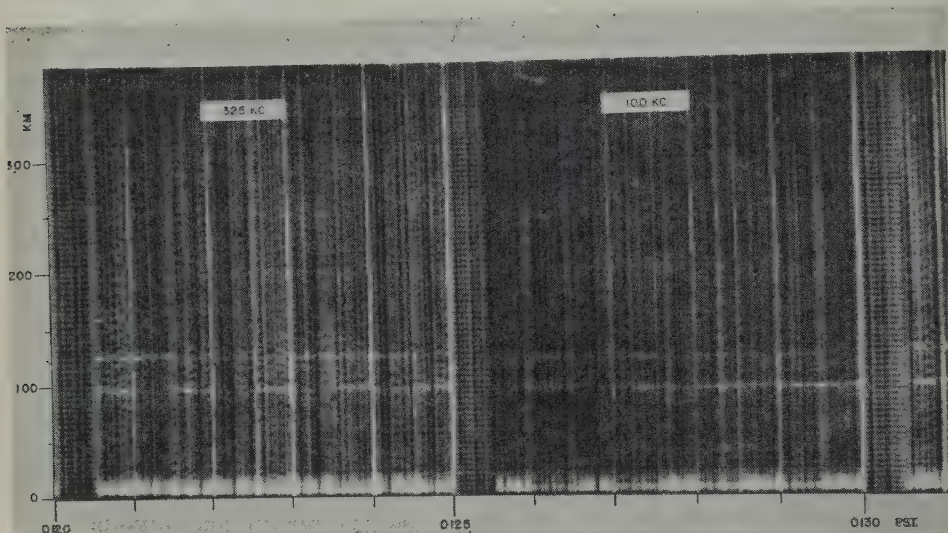

Fig. 6—Original $h't$ record, 3 October 1950, 01^h 20^m-01^h 30^m PST, showing effect of change in frequency on the virtual heights of split *E*-echoes. Lower trace at 97 km on 100 kc and about 3 km higher on 325 kc, while upper trace appears at 122 km on 100 kc and about 5 km higher on 325 kc.

For the fourth explanation we return to our original hypothesis, which is the presence of a multiplicity of ionized strata, each of which produces reflections over a relatively limited range of heights, say, one or two kilometers. Although split reflections are observed simultaneously on one carrier frequency, it cannot be said on the basis of single-frequency data that the lower strata are semi-transparent at that frequency. The wide spectrum of the transmitted pulse and the wide band-width of the receiver might permit observation of several opaque layers simultaneously if the critical frequencies of the lower ones lay within the band of frequencies occupied by the pulse. This possibility has been checked by recording split reflections alternately on two frequencies (100 and 325 kc) whose difference is much larger than the band-width (50 kc) used. An example is shown in Figure 6. Two traces are observed on each frequency and the change in height of each with frequency is small compared with the separation of the traces, thus demonstrating that the splitting of the observed pulse is not the result of the relatively wide spectrum used.

We must now inquire whether our multiple-strata hypothesis is consistent with the observed effects. An important necessary condition required by the multiple-strata explanation is that separation of echoes in the multiple structure should not increase with order. Thus, if two echoes are observed at heights h_1 and h_2 ,

then the first multiple group should show echoes at $2h_1$, $h_1 + h_2$, and $2h_2$; the second multiple group at $3h_1$, $2h_1 + h_2$, $h_1 + 2h_1$, and $3h_2$; etc. . . This characteristic has been observed in nearly all cases where multiples were present. An example is shown in Figure 4*A* and *B*, where it is easily seen that the proper combination heights are present. Although it has been concluded that magneto-ionic separation alone cannot account for the splitting of night-time *E*-region echoes, the observed polarization effects, suggestive of magneto-ionic splitting, are not inconsistent with the multiple-strata hypothesis.

The multiple-strata explanation suggested by these results appears to be in no way inconsistent with the results of other experimenters described above in section II. In particular, it is observed that the splitting of long enduring meteor-echoes found by McKinley and Millman [9] may result from the enhancement or prolongation of meteoric ionization at the particular heights from which the low-frequency echoes are returned. Further information on this point should be obtained by correlating the two types of experiment.

IV—POSSIBLE MODELS OF THE LOWER IONOSPHERE

Although observational data appear to require a multiplicity of layers, they do not indicate the exact form or forms of these layers. It is interesting, however, to speculate as to the forms these layers might take. Three models are suggested, as follows:

(*a*) Thin layers of ionization not more than a few kilometers thick, uniform in horizontal planes, capable of transmitting as well as reflecting appreciable amounts of energy

(*b*) Horizontal layers of flat "pancake-like" clouds of ionization, of such distribution that some energy is reflected from the clouds and some leaks through the spaces between clouds

Evidently either of these models could equally well explain the observed behavior. Both possibilities, however, raise difficult questions regarding the mechanism of formation of such layer structures.

(*c*) It has been suggested by H. G. Booker [10] that scattering due to small irregularities in a thick layer may produce effects similar to those to be expected in a thin layer. While the formation of such a layer is more easily visualized, one must await further theoretical study of the behavior of such a region before confirming its agreement with the many aspects of the observed data.

Whatever the form of the layer selected, it must be explainable on physical grounds. Although as yet there is no theory which explains the formation of such multiple strata, three general mechanisms are suggested, as follows: (*i*) Variation with height of the susceptibility of the region to ionization; (*ii*) variation of the intensity of the ionizing agency with height; and (*iii*) height variation in the turbulence of ionization in a single layer, such that only at certain levels is there sufficient turbulence to cause reflection on a scattering basis.

The ideas suggested above can provide a basis for further work, both experimental and theoretical, which is needed to determine the characteristics of and processes in the lower ionosphere.

V—CONCLUSIONS

The splitting of pulse echoes reflected from the night-time ionosphere at frequencies of 100 and 325 kc has been analyzed with respect to four suggested explanations. The first three, (1) off-vertical reflection, (2) magneto-ionic splitting, and (3) partial reflection in a single layer, are found to have serious drawbacks and are therefore considered as being highly unlikely. The observational data tend to confirm the fourth explanation that splitting is due to the presence of a multiplicity of reflecting strata lying in the height range of about 90 to 130 km, the lower strata being semi-transparent to low-frequency radio waves, and producing reflections over a relatively small range of heights. Systematic differences in the polarizations of split echoes are often observed; lower components tend to be linear in a north-south direction, while upper components tend to be linear in an east-west direction. It is concluded that this polarization effect is not inconsistent with the multiple-strata explanation and suggests that two magneto-ionic modes are reflected. The possible forms and causes of multiple strata are considered. The need for further experimentation is emphasized.

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WAVE PACKETS, THE POYNTING VECTOR, AND ENERGY FLOW: PART I—NON-DISSIPATIVE (ANISOTROPIC) HOMOGENEOUS MEDIA

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ABSTRACT

Two methods for finding the energy velocity in electromagnetic propagation are in common use, the one employing wave packets and the other involving the Poynting vector. However, there appear to have been few attempts to correlate the results, especially for propagation through media of complex natures. The present paper, which is the first of a series, deals only with non-dissipative media, in which the effective permittivity matrix, ϵ_{pq}, is Hermitian, that is, $\epsilon_{pq} = \tilde{\epsilon}_{qp}$. For such media, it is shown that the directions given by the two methods are the same, and hence either may be used for the direction of energy flow.

1—*Introduction*

In simple media (non-dispersive, non-dissipative, isotropic, homogeneous) the energy velocity associated with infinite homogeneous plane waves is in the direction of phase propagation and has magnitude equal to the phase speed. In some texts this is considered as apparent, while in others it is justified by dividing the Poynting vector (that is, the energy flow) by the energy density to obtain the required quantity.

In dispersive media the energy speed is given more accurately by the group speed—the speed of a group of waves of differing frequencies which are superimposed to build a pulse. Havelock [see 1 of "References" at end of paper] gives an excellent discussion of group speed as it was developed by Stokes [2], Rayleigh [3], Kelvin [4], and others. This speed is shown by him, and in more detail by White [5], to be the same as that calculated from the Poynting vector for propagation through media having simplified (no absorption) Lorentz dispersion. The group speed fails to give the energy speed near resonance frequencies, however, and a more delicate treatment is then required. For such cases, see Stratton [6] or the original work by Sommerfeld and Brillouin [7].

In treating anisotropic non-dissipative media, the common procedure is to obtain the direction of energy flow from the Poynting vector. Drude [8] uses this method, and goes on to determine the energy velocity on the basis that its component in the direction of phase propagation is the phase speed. The velocity

vector thus obtained is equal to the Poynting vector divided by the energy density (see Stratton [6], p. 342).

When a dispersive anisotropic medium is to be considered, generalizations of both the above methods are available. Booker [9] discusses the propagation of a wave packet in a particular medium of this type—a conducting medium with a constant magnetic field applied (for example, the ionosphere). The packet is made by superimposing waves traveling in different directions, as well as having different frequencies. On the other hand, Scott [10, 11] uses the mean Poynting vector to obtain the direction of energy flow in the same type of medium. When collisional damping is included in this theory, Scott obtains a component of energy flow in the direction perpendicular to the magnetic field and to the phase normal, whereas Booker obtains none. This discrepancy led to the present investigation of the equivalence of the wave packet and Poynting vector approaches to energy flow.

The discussion in the present paper is confined to homogeneous, non-magnetic media having a Hermitian effective permittivity. This Hermitian condition is frequently obtained; it occurs in non-conducting crystals and in media like the ionosphere if collision is not included (see Appendix). It is the condition that no net energy be lost from the electromagnetic field, that is, that the medium is non-dissipative, as shown in section 2 below. Unfortunately, the condition is not obtained when damping is included—the case which gave rise to the present investigation. In cases of absorbing and non-homogeneous media, the problem is much more complex, both physically and mathematically. It is hoped that these may be considered in subsequent papers.

2—*Effective permittivity*

In simple material media, we consider the total effective current $\mathbf{J} + (\partial/\partial t)\mathbf{D}$ to consist of three parts, as follows: The conduction current $\mathbf{J}$, the true part of the displacement current $\partial/\partial t\ (\kappa - 1)\ \mathbf{E}$, and the fictitious part $(\partial/\partial t)\mathbf{E}$. The first two are usually distinguished by the nature of the charges moving in them—whether they are "unbound" or "bound." A second distinguishing feature is equivalent in the simple case—whether the current is in phase or out of phase with an oscillating electric field. This distinction leads to another (equivalent) one—whether network is or is not done on the charges in a complete cycle. This last feature is possibly the most important in practice, and may be generalized for conducting anisotropic media as below.

We shall consider electric waves varying cyclicly with time at a frequency $kc/2\pi$, and an anisotropic medium having the linear relations

$$D_p = \kappa_{pq}E_q \qquad J_p = \sigma_{pq}E_q \qquad (p,\ q = 1,\ 2,\ 3) \qquad (1)$$

Here D_p , J_p , and E_p are the "p^{th}" components of the complex displacement, current, and electric vectors corresponding to the real vectors. The dielectric and conductivity matrix components κ_{pq} and σ_{pq} will be complex, in general (see, for example, the development for the ionosphere given in Appendix). In all electromagnetic equations, $\mathbf{D}$ and $\mathbf{J}$ appear in the combination $\mathbf{D} + \int \mathbf{J}\ dt$, which may be replaced by an "effective displacement" vector

$$\mathbf{D}_{\text{eff}} = \mathbf{D} + \frac{i}{kc}\,\mathbf{J} \qquad \text{(2)}$$

in the present circumstances. It is the real part of (2) which enters physically valid equations, of course.

By (1) and (2) we can introduce the "effective permittivity matrix"

$$\epsilon_{pq} = \kappa_{pq} + \frac{i}{kc}\,\sigma_{pq} \qquad \text{(3)}$$

such that

$$(D_{\text{eff}})_p = \epsilon_{pq} E_q \qquad \text{(4)}$$

As with all matrices, ϵ_{pq} can be written as the sum of a Hermitian and an anti-Hermitian matrix

$$\epsilon_{pq} = h_{pq} + a_{pq} \qquad \text{(5)}$$

where $h_{pq} = \tilde{h}_{qp}$ and $a_{pq} = -\tilde{a}_{qp}$ (the symbol $\tilde{}$ is used to indicate the complex conjugate throughout).

Poynting's equation, in terms of the real field vectors, may be written

$$-\frac{c}{4}\int_S [(\mathbf{E} + \tilde{\mathbf{E}}) \times (\mathbf{H} + \tilde{\mathbf{H}})]\cdot d\mathbf{s}$$

$$= \frac{1}{4}\int_V \left[(\mathbf{E} + \tilde{\mathbf{E}})\cdot\frac{\partial}{\partial t}(\mathbf{D}_{\text{eff}} + \tilde{\mathbf{D}}_{\text{eff}}) + (\mathbf{H} + \tilde{\mathbf{H}})\cdot\frac{\partial}{\partial t}(\mathbf{H} + \tilde{\mathbf{H}})\right] dv \qquad \text{(6)}$$

as can be readily verified. The left side is usually taken as the flow of energy into a volume V, and the right side as the rate of increase of energy inside V. By a well-known theorem for time-averages of products such as we have here (see Stratton [6], sec. 2.20), the mean rate of increase of this energy is the real part of

$$\frac{1}{2}\int_V \left[\mathbf{E}\cdot\frac{\partial}{\partial t}\tilde{\mathbf{D}}_{\text{eff}} + \mathbf{H}\cdot\frac{\partial}{\partial t}\tilde{\mathbf{H}}\right] dv \equiv \frac{1}{2}\int_V ikc[E_p(\tilde{h}_{pq} + \tilde{a}_{pq})\tilde{E}_q + \mathbf{H}\cdot\tilde{\mathbf{H}}]\, dv \qquad \text{(7)}$$

Now $E_p\tilde{h}_{pq}\tilde{E}_q$ and $\mathbf{H}\cdot\tilde{\mathbf{H}}$ are real, while $E_p\tilde{a}_{pq}\tilde{E}_q$ is pure imaginary, hence this mean rate of increase of energy is simply

$$\frac{ikc}{2}\int_V E_p\tilde{a}_{pq}\tilde{E}_q\, dv \qquad \text{(8)}$$

This corresponds to the simpler expression $\sigma/2 \int \mathbf{E}\cdot\tilde{\mathbf{E}}\, dv$ which occurs for isotropic conducting media. Any net gain of energy will appear as heat and so is lost (dissipated) from the electromagnetic field.

While (8) may vanish for special fields, it will not do so in general unless all $a_{pq} = 0$—that is, unless the effective permittivity is Hermitian. This gives the condition that the medium will be non-dissipative.

It may be noted in passing that, for energy considerations, it is h_{pq} and $-ikc\, a_{pq}$—not κ_{pq} and σ_{pq}—which appear as generalizations of κ and σ. The artificiality of the distinction of currents as "conduction" or "true displacement" is apparent in the treatment for the ionosphere.

3—*General mathematics*

We begin with Maxwell's curl equations, in Heaviside-Lorentz units

$$\operatorname{curl} \mathbf{H} = \frac{1}{c}\mathbf{J} + \frac{1}{c}\frac{\partial}{\partial t}\mathbf{D} \qquad \operatorname{curl} \mathbf{E} = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{H} \qquad (9)$$

and assume the constitutive relations (1) to (5) above, putting $a_{pq} = 0$ for the present paper. Plane homogeneous waves having wave normal in the $\lambda\mu\nu$ direction and frequency $kc/2\pi$ will have complex components of all vectors proportional to exponential $i\Phi$, where

$$\Phi \equiv k[n(\lambda x + \mu y + \nu z) - ct] \qquad (10)$$

For such waves, all field components except the E_p's may now be eliminated from the equations available, yielding

$$(h_{11} - n^2 + n^2\lambda^2)E_1 + (h_{12} + n^2\lambda\mu)E_2 + (h_{13} + n^2\lambda\nu)E_3 = 0 \qquad (11)$$

$$(h_{21} + n^2\lambda\mu)E_1 + (h_{22} - n^2 + n^2\mu^2)E_2 + (h_{23} + n^2\mu\nu)E_3 = 0 \qquad (12)$$

$$(h_{31} + n^2\lambda\nu)E_1 + (h_{32} + n^2\mu\nu)E_2 + (h_{33} - n^2 + n^2\nu^2)E_3 = 0 \qquad (13)$$

For non-vanishing fields, the determinant of the bracketed expressions must vanish, yielding a quadratic in n^2 (the terms in n^6 cancel), as follows:

$$f \equiv \alpha_4 n^4 + \alpha_2 n^2 + \alpha_0 = 0 \qquad (14)$$

where

$$\alpha_4 = \lambda^2 h_{11} + \mu^2 h_{22} + \nu^2 h_{33} + \mu\nu(h_{23} + h_{32}) + \nu\lambda(h_{31} + h_{13}) + \lambda\mu(h_{12} + h_{21})$$

$$\begin{aligned}\alpha_2 = {} & (1 - \lambda^2)(h_{32}h_{23} - h_{22}h_{33}) + \mu\nu(h_{31}h_{12} + h_{13}h_{21} - h_{11}h_{23} - h_{11}h_{32}) \\ & + (1 - \mu^2)(h_{31}h_{13} - h_{33}h_{11}) + \nu\lambda(h_{12}h_{23} + h_{21}h_{32} - h_{22}h_{31} - h_{22}h_{13}) \\ & + (1 - \nu^2)(h_{12}h_{21} - h_{11}h_{22}) + \lambda\mu(h_{13}h_{32} + h_{23}h_{31} - h_{33}h_{12} - h_{33}h_{21})\end{aligned}$$

$$\alpha_0 = h_{11}h_{22}h_{33} - h_{11}h_{23}h_{32} - h_{22}h_{31}h_{13} - h_{33}h_{12}h_{21} + h_{23}h_{31}h_{12} + h_{32}h_{13}h_{21}$$

For convenience of reference with subsequent papers, we may add the definitions $\alpha_3 = \alpha_1 = 0$ and write (14) as

$$\Sigma \alpha_r n^r = 0 \qquad (r = 0, \cdots, 4)$$

We shall want for our discussion the partial derivatives $\beta_r = (\partial\alpha_r/\partial\lambda)_\mu$ and $\gamma_r = (\partial\alpha_r/\partial\mu)_\lambda$ remembering that $\lambda^2 + \mu^2 + \nu^2 = 1$, from which $(\partial\nu/\partial\lambda)_\mu = -\lambda/\nu$ and $(\partial\nu/\partial\mu)_\lambda = -\mu/\nu$. Our discussion will be no less general if we consider phase propagation as occurring in the z-direction, that is, $\lambda = \mu = 1 - \nu = 0$. This condition greatly simplifies the required quantities; under it we obtain

$$\alpha_4 = h_{33} \qquad (15)$$

$$\alpha_2 = h_{31}h_{13} + h_{23}h_{32} - h_{33}(h_{11} + h_{22}) \qquad (16)$$

$$\alpha_0 = h_{11}h_{22}h_{33} - h_{11}h_{23}h_{32} - h_{22}h_{31}h_{13} - h_{33}h_{12}h_{21} + h_{23}h_{31}h_{12} + h_{32}h_{13}h_{21} \qquad (17)$$

$$\beta_4 = h_{31} + h_{13} \qquad (18)$$

$$\beta_2 = h_{12}h_{23} + h_{21}h_{32} - h_{22}(h_{31} + h_{13}) \qquad (19)$$

$$\gamma_4 = h_{23} + h_{32} \qquad (20)$$

$$\gamma_2 = h_{31}h_{12} + h_{13}h_{21} - h_{11}(h_{23} + h_{32}) \qquad (21)$$

$$\alpha_3 = \alpha_1 = \beta_3 = \beta_1 = \beta_0 = \gamma_3 = \gamma_1 = \gamma_0 = 0 \quad \text{always} \qquad (22)$$

The discriminant of the quadratic (14) can be calculated from (15) to (17); it can be reduced to the following:

$$\alpha_2^2 - 4\alpha_4\alpha_0 = [h_{31}h_{13} - h_{23}h_{32} - h_{33}(h_{11} - h_{22})]^2 + 4(h_{23}h_{31} - h_{21}h_{33})(h_{32}h_{13} - h_{12}h_{33}) \qquad (23)$$

Due to the Hermitian nature of h_{pq}, all the α's, β's, and γ's are real. Moreover, the discriminant (23) is non-negative, hence n^2 as given by (14) is real. It follows that n is either pure real or pure imaginary—the former being the only case of present interest.

We shall also want the relative components of **E**, which may be obtained from any two of (11) to (13). On setting $E_q = P_qA$ and using the last two of these with $\lambda = \mu = 1 - \nu = 0$, we obtain

$$P_1 = h_{22}h_{33} - h_{23}h_{32} - n^2h_{33} \qquad (24)$$

$$P_2 = h_{31}h_{23} - h_{21}h_{33} \qquad (25)$$

$$P_3 = h_{21}h_{32} - h_{31}h_{22} + n^2h_{31} \qquad (26)$$

and it may be observed that

$$h_{31}P_1 + h_{32}P_2 + h_{33}P_3 = 0 \qquad (27)$$

Due to the Hermitian nature of h_{pq}, it is readily seen that P_1 is real; it may also be shown that

$$P_1^2 - P_2\tilde{P}_2 = [h_{31}h_{13} - h_{23}h_{32} - h_{33}(h_{11} - h_{22})]P_1 \qquad (28)$$

a relation that will be useful later. [This is most easily shown by calculating the left-hand side and removing the term in n^4 by use of (14) to (17).]

4—Wave packets

A wave packet consists of the superposition of varying amounts of infinitely extended plane waves having varying frequencies (hence varying k) and varying directions of phase propagation (hence varying λ, μ, ν). By proper choice of the complex amplitude factors, the resulting interference can be made to localize the field to a small region. The packet can properly be expressed as a Fourier triple integral, and ideally such an integral could be evaluated at different times, thus giving the progress and distortion of the packet. Practically, however, we confine our attention to frequencies and directions near some "central" values, and follow the point of the maximum amplitude.

We shall follow the common (though lax) development of packet velocity, leaving till a later paper a more critical investigation which will be necessary for generalization. Our packet will consist of waves having unit amplitude if $k_0 - K < k < k_0 + K$, $-L < \lambda < L$, $-M < \mu < M$, and zero amplitude otherwise. On the grounds that K, L, and M are to be kept small, $\Phi(k, \lambda, \mu)$ as given by (10) is expanded in a Taylor series and only the first four terms are retained, as follows:

$$\Phi = \Phi_0 + \Phi_k(k - k_0) + \Phi_\lambda \lambda + \Phi_\mu \mu \qquad (29)$$

where $\Phi_0 = \Phi(k_0, 0, 0)$ and the partial derivatives $\Phi_k \Phi_\lambda \Phi_\mu$ are evaluated at k_0, 0, 0. Our packet then contains fields proportional to

$$F = \int_{-K}^{K} \int_{-L}^{L} \int_{-M}^{M} \epsilon^{i(\Phi_0 + \Phi_k \kappa + \Phi_\lambda \lambda + \Phi_\mu \mu)} \, d\kappa \, d\lambda \, d\mu \qquad (30)$$

where κ is written for $k - k_0$. The integral is readily evaluated; on substituting for Φ_κ, Φ_λ, and Φ_μ it becomes

$$F = 8\epsilon^{i\Phi_0} \frac{\sin\left[\left(\frac{\partial kn}{\partial k} z - ct\right)K\right]}{\left(\frac{\partial kn}{\partial k} z - ct\right)} \frac{\sin\left[k(nx + n_\lambda z)L\right]}{k(nx + n_\lambda z)} \frac{\sin\left[k(ny + n_\mu z)M\right]}{k(ny + n_\mu z)} \qquad (31)$$

This obviously has maximum amplitude at the point given by

$$\frac{\partial kn}{\partial k} z - ct = 0 \qquad (32)$$

$$nx + n_\lambda z = 0 \qquad (33)$$

$$ny + n_\mu z = 0 \qquad (34)$$

These are the equations of motion of the wave packet. From (32), the z-component of the velocity is $c/(\partial kn/\partial k)$, while from (33) and (34), the direction of motion is given by the ratios $n_\lambda : n_\mu : -n$.

Now (14) is of the form $f(\lambda, \mu, k, n) = 0$, since $\nu = \nu(\lambda, \mu)$ and, in general, $h_{pq} = h_{pq}(k)$. Then

$$n_\lambda = -f_\lambda / f_n \qquad n_\mu = -f_\mu / f_n \qquad (35)$$

and the direction of motion can be given by the ratios $f_\lambda : f_\mu : nf_n$. On substituting for the partial derivatives, this reduces to

$$\beta_4 n^2 + \beta_2 : \quad \gamma_4 n^2 + \gamma_2 : \quad 4\alpha_4 n^2 + 2\alpha_2 \qquad (36)$$

5—*Poynting vector*

The direction of mean energy flow is that of the real part of the complex Poynting vector $\mathbf{S} = c/2\, \mathbf{E} \times \tilde{\mathbf{H}}$ (see Stratton [6]). Using the relations in section 3 and setting $\lambda = \mu = 1 - \nu = 0$, we get

$$H_1 = -nE_2 \qquad H_2 = nE_1 \qquad H_3 = 0 \qquad (37)$$

Then

$$\mathbf{S} = \frac{c}{2} A\tilde{A}\, [-P_3\tilde{n}\tilde{P}_1\,, \quad -P_3\tilde{n}\tilde{P}_2\,, \quad P_1\tilde{n}\tilde{P}_1 + P_2\tilde{n}\tilde{P}_2]$$

where we have set $E_q = P_q A$ as above. The direction of mean energy flow is then given by

$$\text{R.P.}\,(-\tilde{n}\tilde{P}_1 P_3) \quad : \quad \text{R.P.}\,(-\tilde{n}\tilde{P}_2 P_3) \quad : \quad \text{R.P.}\,(\tilde{n}P_1\tilde{P}_1 + \tilde{n}P_2\tilde{P}_2)$$

where R.P. denotes the real part of the bracketed expression.

In the present case, with n and P_1 real, these ratios become

$$-P_1(P_3 + \tilde{P}_3) \quad : \quad -(\tilde{P}_2 P_3 + P_2\tilde{P}_3) \quad : \quad 2(P_1^2 + P_2\tilde{P}_2) \qquad (38)$$

6—*Conditions of equality*

The conditions that the two directions determined in sections 4 and 5 should be the same are obviously

$$\frac{P_1(P_3 + \tilde{P}_3)}{\beta_4 n^2 + \beta_2} = \frac{\tilde{P}_2 P_3 + P_2\tilde{P}_3}{\gamma_4 n^2 + \gamma_2} = -\frac{P_1^2 + P_2\tilde{P}_2}{2\alpha_4 n^2 + \alpha_2} \qquad (39)$$

from (36) and (38). These conditions can be reduced directly, by substitution of (14) to (26), into conditions on the h_{pq}'s. However, a much less laborious method is the following: P_3 and $\tilde{P}_3$ can be eliminated by means of (27). After slight reduction, (39) then becomes

$$\left.\begin{aligned} &\frac{P_1^2 + P_1\left[\dfrac{h_{32}P_2 + h_{23}\tilde{P}_2}{h_{13} + h_{31}}\right]}{h_{22} - n^2 - \left[\dfrac{h_{12}h_{23} + h_{32}h_{21}}{h_{13} + h_{31}}\right]} = \frac{P_2\tilde{P}_2 + P_1\left[\dfrac{h_{13}P_2 + h_{31}\tilde{P}_2}{h_{23} + h_{32}}\right]}{h_{11} - n^2 - \left[\dfrac{h_{31}h_{12} + h_{21}h_{13}}{h_{23} + h_{32}}\right]} \\ &\qquad = \frac{P_1^2 + P_2\tilde{P}_2}{h_{22} - h_{11} - 2n^2 - \left[\dfrac{h_{31}h_{13} + h_{23}h_{32}}{h_{33}}\right]}. \end{aligned}\right\} \qquad (40)$$

If these conditions (40) be written as $a/b = c/d = e/f$, then an elementary theorem tells us that

$$\frac{a}{b} = \frac{a - c}{b - d} = \frac{2a - e}{2b - f} \qquad (41)$$

etc. The conditions (41) are equivalent to (40), and happen to be much more easily treated. This is because the numerators of the second and third fractions in (41) contain $P_1^2 - P_2\tilde{P}_2$, which can be replaced [using (28)] by a term proportional to P_1, and other terms proportional to P_1. All numerators in (41) will then have a common factor P_1, which may be canceled. [If $P_1 = 0$, equations (41) are automatically satisfied.] Moreover, n^2 is not present in the denominators of the second and third fractions, nor in their numerators once P_1 is canceled. Direct substitution for P_1, P_2, and $\tilde{P}_2$ from (24) and (25), at this point, reduces the conditions to the identity $h_{33} = h_{33} = h_{33}$. That is, the conditions (39) are always satisfied under the conditions assumed in this paper, namely, $\epsilon_{pq} = h_{pq} = \tilde{h}_{qp} = \tilde{\epsilon}_{qp}$, and n is pure real. Due to the breakdown of the concepts involved, there is little purpose in considering the case where n is pure imaginary, though this is easily done. The two directions are then not, in general, the same.

7—*Discussion and conclusions*

No attempt has been made to calculate the "Poynting velocity"—that is, the velocity obtained on dividing the Poynting vector by the energy density. It can be seen, however, that in general the speed which would result will not equal the speed resulting from the packet concept. This is because the latter depends on the dispersion n_k, whereas the former does not. Only in media having the proper type of $n = n(k)$ relation will the speeds be the same. This discrepancy is usually attributed to the error of treating the energy of infinite waves in localized pieces (whose speed would be the flow divided by the density), no matter how fruitful this concept is in non-dispersive media. It has been shown, however, that no such discrepancy occurs in the direction given by the packet velocity and the mean Poynting vector, and hence we may state as the conclusion of the present investigation the following: When plane homogeneous waves are propagated through a homogeneous non-absorbing medium, the concept of packet velocity and the mean Poynting vector give agreement in determining the direction in which energy is assumed to flow. Either may be used for this purpose.

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Appendix

In the ionosphere it is usual to consider $\kappa_{pq} = \delta_{pq}$ (the Kronecher delta), that is, $\mathbf{D} = \mathbf{E}$, and to consider the motion of the free charges as contributing to a true current $\mathbf{J}$. In this current, only the electrons (mass m, charge e) are considered, since the motion of other (heavy) charged particles under the action of the radiation field will be much less. The equation of motion of the electrons is approximately

$$m\dot{\mathbf{V}} = e\mathbf{E} + \frac{e}{c}\mathbf{V} \times \mathbf{H}_0 - \nu m\mathbf{V} \qquad (42)$$

where $\mathbf{V}$ is their velocity, $\mathbf{H}_0$ is the earth's magnetic field, ν is the collision frequency. Here the effect of the radiation magnetic field is neglected, it being small compared to that of the radiation electric field so long as V/c is small. Equation (42) yields the three scalar equations

$$E_1 = \left(\frac{\nu m - ikcm}{e}\right) V_1 - \frac{1}{c} H_z V_2 + \frac{1}{c} H_y V_3 \qquad (43)$$

$$E_2 = \frac{1}{c} H_z V_1 + \left(\frac{\nu m - ikcm}{e}\right) V_2 - \frac{1}{c} H_x V_3 \qquad (44)$$

$$E_3 = -\frac{1}{c} H_y V_1 + \frac{1}{c} H_x V_2 + \left(\frac{\nu m - ikcm}{e}\right) V_3 \qquad (45)$$

where $H_x H_y H_z$ are the components of the earth's field, and we have taken $V \propto \exp -ikct$. The coefficients on the right-hand side have determinant

$$\Delta = g^3 + \frac{g}{c^2} H_0^2 \qquad (46)$$

where

$$g = \frac{\nu m - ikcm}{e} \quad \text{and} \quad H_0 = |\,\mathbf{H}_0\,| = \sqrt{H_x^2 + H_y^2 + H_z^2}$$

The equations (43) to (45) are then readily inverted, giving

$$\Delta c^2 V_1 = (H_x^2 + c^2 g^2)E_1 + (H_x H_y + cgH_z)E_2 + (H_z H_x - cgH_y)E_3$$

$$\Delta c^2 V_2 = (H_x H_y - cgH_z)E_1 + (H_y^2 + c^2 g^2)E_2 + (H_y H_z + cgH_x)E_3$$

$$\Delta c^2 V_3 = (H_z H_x + cgH_y)E_1 + (H_y H_z - cgH_x)E_2 + (H_z^2 + c^2 g^2)E_3$$

Now we have for current $\mathbf{J} = Ne\mathbf{V}$, where N is the electron density. Hence $J_p = NeV_p = \sigma_{pq}E_q$, from which the conductivity matrix is

$$\sigma_{pq} = \frac{Ne}{\Delta c^2} \left\| \begin{matrix} H_x^2 + c^2 g^2 & H_x H_y + cgH_z & H_z H_x - cgH_y \\ H_x H_y - cgH_z & H_y^2 + c^2 g^2 & H_y H_z + cgH_x \\ H_z H_x + cgH_y & H_y H_z - cgH_x & H_z^2 + c^2 g^2 \end{matrix} \right\| \qquad (47)$$

The effective permittivity matrix is, then,

$$\epsilon_{pq} = \delta_{pq} + \frac{i}{kc}\,\sigma_{pq} \qquad (48)$$

with σ_{pq} given by (47).

In the case when collision is neglected, $\nu = 0$, g and Δ are pure imaginary. The i in (48) and that of Δ in (47) will cancel, and it is readily seen that the matrix factor in (47) is Hermitian; hence the effective permittivity ϵ_{pq} is also.

(*Note*: It is assumed here that N is independent of the radiation field **E**. This is not true, in fact, due to "bunching" of electrons, but the correction is a small one and is not usually considered. See J. Feinstein, J. Geophys. Res., **55**, 161, 1950.)

MAGNETIC POLARIZATION OF TERTIARY ROCKS IN JAPAN

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ABSTRACT

The direction and intensity of magnetic polarization of Tertiary sediments and volcanic rocks in various localities in Japan were measured. Samples were collected from Upper Miocene to the recent beds in Kanto, Toyama, and Kinki districts. A series of samples of sedimentary strata in Kanto, ranging from the lowest bed of Upper Miocene to the uppermost layer of Pleistocene, were examined especially in detail. The declination of magnetic polarization of the whole section, more than 4,000 meters in thickness, is nearly the same as that of the present earth's magnetic field, while the inclination gradually decreases from the mean value of 25° in Miocene to 50° in Pleistocene beds.

The reverse polarization was found only in the cases of pyroclastic (volcanic) sediments in the upper part of Upper Neogene and volcanic lavas of the same geological age in Kinki district.

I—*Apparatus and measurement*

The apparatus used in the present experiment was a modified astatic magnetometer [see 1 of "References" at end of paper]. A specimen to be measured is rotated at a place just below the lower magnet of the astatic magnetometer with a uniform angular velocity which is equal to the phase velocity of the proper oscillation of the magnet system. By means of this method, the direction of magnetic polarization of greater than 10^{-7} c.g.s./cm^3 in intensity could be determined with errors less than 1°, the limit of detection being 10^{-9} c.g.s./cm^3.

II—*Sampling and corrections*

The errors of sampling were as small as 1° in ordinary rocks. The direction of polarization was corrected for the amount of tilt of the strata. However, the angles of rotation of bedding planes around their normals and the angles of original dip of the strata at the time of depositions were not ascertained. Neither of these angles was taken into consideration as correction, because both were small.

Wulff's net of stereographic projection was used to correlate the observed orientation of polarization into the geographical coordinate, and also to correct for the tilt of the bedding plane. Compared with the errors due to sampling,

observation, and projection, each of which is less than 1°, the error in the correction for the tilt of layer would not be so small, because the determination of bedding planes was not so accurate owing to the uneven and obscure surfaces.

III—*Results obtained in Bôsô Peninsula*

T. Nagata has already studied in detail [2,3] the uppermost strata of Pleistocene sediments in Bôsô Peninsula. According to his results, the direction of the past earth's magnetic field was nearly the same as the present field. In order to extend the range of sampling from Pleistocene back to Miocene, the writer has collected the samples from Upper Miocene to the recent beds in the Bôsô Tertiary sedimentary basin. It is believed that continuous deposition took place in a rather shallow sea in this geosyncline, except for a time interval between Miocene and Pliocene.

The thickness of the accumulated sediments is evaluated as more than 6,000 meters. The whole sedimentary section has been lifted above sea level and tilted simultaneously northward about 3° to 15°. The actual sampling was made from the northern parts of the Peninsula. Since the Pleistocene layers are tilted northward about 8° and the Pliocene layers about 12°, the sampling every 220 meters and 120 meters in horizontal distance on the Pleistocene and Pliocene zones, respectively, correspond to sampling every 30 meters in depth. In the Miocene, however, there exists a marked anticlinal fold whose axis is parallel to the direction of N 70° W, while the dip of bedding plane varies from 0° to 50° on both sides of the axis. Hence the samples were taken off here every 30 meters in thickness of the layer after a few careful examinations of strata.

Thus, the mean sampling interval throughout the whole layers was about 30 meters.

The direction of magnetic polarization, intensity, susceptibility, and other magnetic characters of 150 samples thus collected were determined in the laboratory. The directions of magnetic polarization of the sediments are illustrated vectorially in Figure 1, in which the direction of vectors represents the deviation from the geographic meridian, while its length is proportional to the cosine of inclination.

Table 1

Geologic age	Mean declination	Mean inclination	Number of samples
	°	°	
Pleistocene	N 6.2 W	44.8	35
Upper Pliocene	N 4.0 W	39.7	31
Lower Pliocene	N 3.8 W	24.3	16
Miocene	N 1.5 W	28.3	65

In Table 1 are tabulated the mean declination and the mean inclination in each geologic time. In Figure 2, the general change of magnetic polarization with depth is shown.

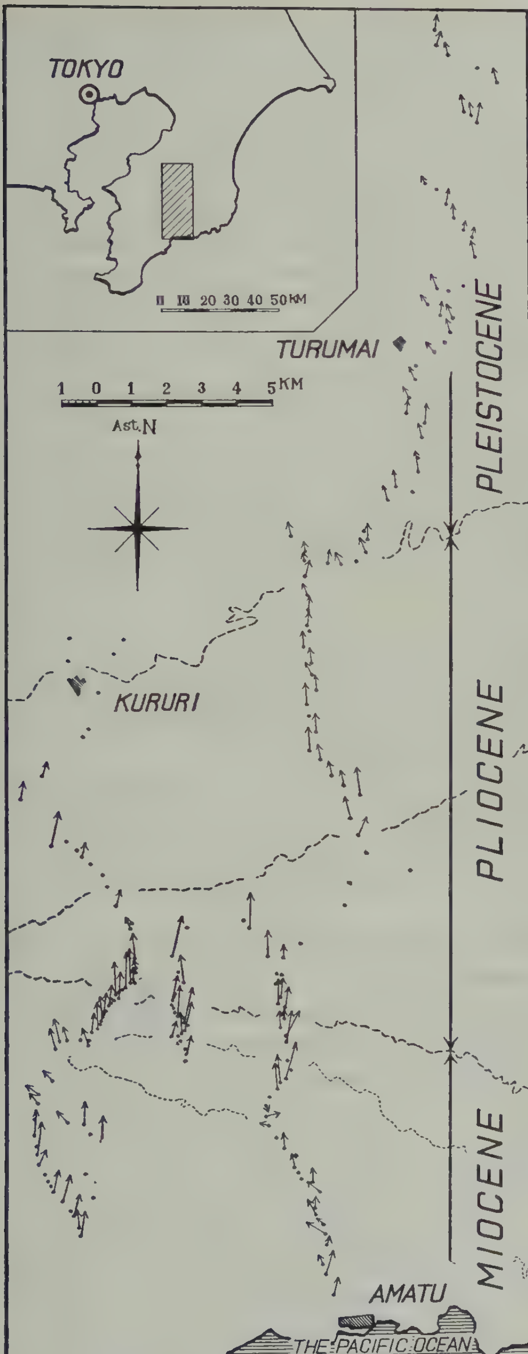
TOKYO
20 30 40 50 KM
TURUMAI
1 0 1 2 3 4 5 KM
Ast. N
KURURI
AMATU
THE PACIFIC OCEAN
PLEISTOCENE
PLIOCENE
MIOCENE

Fig. 1

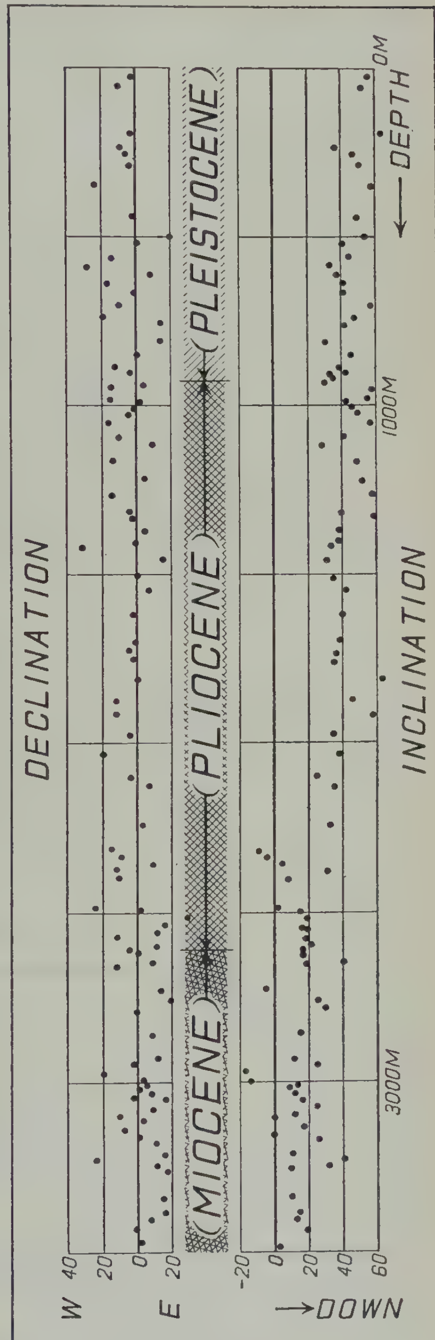
DECLINATION
(PLEISTOCENE)
(PLIOCENE)
(MIOCENE)
INCLINATION
DEPTH
0M
1000M
3000M
W 40 20 0 20 E
-20 0 20 40 60
→DOWN

Fig. 2

As will be seen in Figures 1 and 2, the fluctuations of declination are within the range from N 30° E to N 30° W. It seems that the declination changes gradually from N 1.°5 W in Miocene to N 6.°2 W in Pleistocene. On the other hand, the changes in the mean inclinations are greater than those in the mean declination. The largest value of inclination of about 60° occurs in the uppermost part of the Pleistocene sediments, which belong to the Narita bed. The inclination and declination of the Narita bed obtained here agree well with Nagata's results [3]. However, the Lower Pliocene and the Upper Miocene sediments have smaller inclination; the specimens frequently have their directions of polarization almost parallel to bedding planes, and even the upward dips of about 10° from the bedding plane were observed in several extreme cases, such as in tuffaceous clay and shale of Kiwada, Yasumno, Kiyozumi bed, etc.

Roughly speaking, the change in declinations is less than the change in inclinations throughout the whole Tertiary sediments.

J. W. Graham [4] has studied various sediments in the United States of America and obtained sufficient data to prove the stabilities of polarizations. Since the present data of the sediments of Bôzô Peninsula are not conclusive enough to confirm the stability of polarization, this problem will be studied after more Japanese data are accumulated.

IV—*Results obtained in Kinki district*

The Upper Neogene sediments and volcanic rocks in Kinki district (middle Japan) and Mio-Pliocene rocks in Toyama district (north Japan) were also examined, although the studies are still in a preliminary stage.

Samples in the Kinki district were collected from the so-called "Osaka sedimentary group" and lavas of "Setouti volcanic zone." The Osaka group is composed of sands, gravels, silts, and five sheets of tuff layers deposited in the old Setouti Inland Sea and piled upon the eroded basal surface of granite and gneiss or Paleozoic formations.

Although magnetic polarizations of many of the Osaka group samples were too weak to be measured, their intensity being less than 10^{-8} c.g.s./cm^3, some sandy or silty parts of the sediments could be measured. The directions of magnetic polarizations of these sediments were, roughly speaking, generally normal and nearly the same as that of the present earth's magnetic field, except for those of a tuff which lies in the upper part of the Osaka group. It was found that this tuff layer was magnetized in opposition to the present earth's magnetic field; that is, its north-seeking pole pointed southward and upward. This tuff, usually called the Azuki tuff, is a nearly flat-lying sedimentary bed, free from evidence of any crustal movement. It has been found in several localities around the hilly area near Osaka City.

The specimens of the tuff have been collected from 60 localities in the northern and southern suburbs of Osaka City. Thirty-two of these samples had enough magnetic polarization to be measured.

From the petrological studies made by S. Kokawa, the Azuki tuff has been determined to be an andesitic tuff which has fragmental crystals of rhombic and monoclinic pyroxene, as well as turbid colorless glass flakes with small quantities of magnetites. It has been concluded from geologic evidence that these pyroclastic

materials were derived from the Setouti volcanic zone, in which the volcanoes ejected and distributed the pyroclastic material throughout the Setouti Inland Sea during the deposition of the Osaka group.

Seventy-three lava samples were taken from 12 Setouti lava flows in the Hyôgo, Nara, and Osaka prefectures. Although the greater part of the old volcanoes shows normal polarizations, there are some with reverse polarizations, the directions of which are nearly the same as that of the Azuki tuff. Generally speaking, normal polarizations were found in the earlier and later ejecta, while the reverse polarizations were found in the intermediate eruptions such as the volcanoes Shigi, Hôzanji, Tomeshô, etc. The north-seeking poles of the normal polarizations of the Setouti eruptive lavas are plotted in Figure 4 (on the lower hemisphere), and those of the reverse polarizations are plotted together with the Azuki tuff in Figure 3 (on the upper hemisphere).

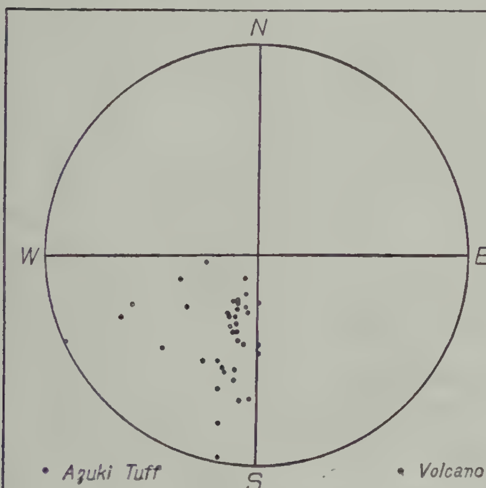

Fig. 3

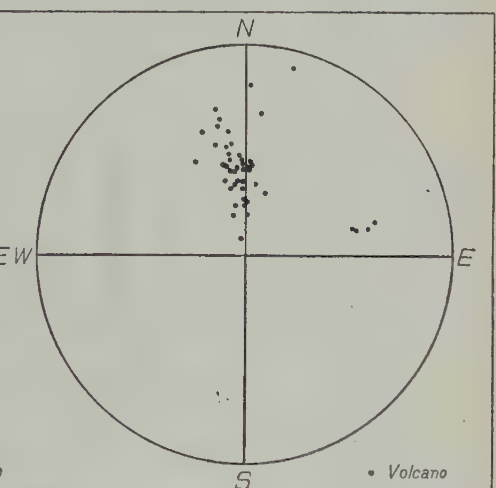

Fig. 4

The volcanic rocks of Shigi, Hôzanji, Tomesho, etc. (reversely polarized lava flows) are all two-pyroxene andesites. It may be a very significant fact that the Azuki tuff is a two-pyroxene andesitic tuff, because two-pyroxene andesites are scarcely seen in the Setouti zone.

In Figure 5, the distributions of Setouti volcanoes and the Azuki tuff are illustrated, and the schematic correlations between Osaka group and volcanic activity of the old Setouti zone are shown in Figure 6.

All the specimens of both sedimentary and volcanic rocks appear as if they have not received many crustal movements and have essentially maintained the initial positions of their depositions in water, and their solidification from magma, respectively.

In order to examine the uniformity of the reverse magnetization, the distribution of vertical magnetic force was measured with a Schmidt magnetometer in the neighborhood of the sampling area. The results of this magnetic survey showed that the lava flow is reversely magnetized as a whole.

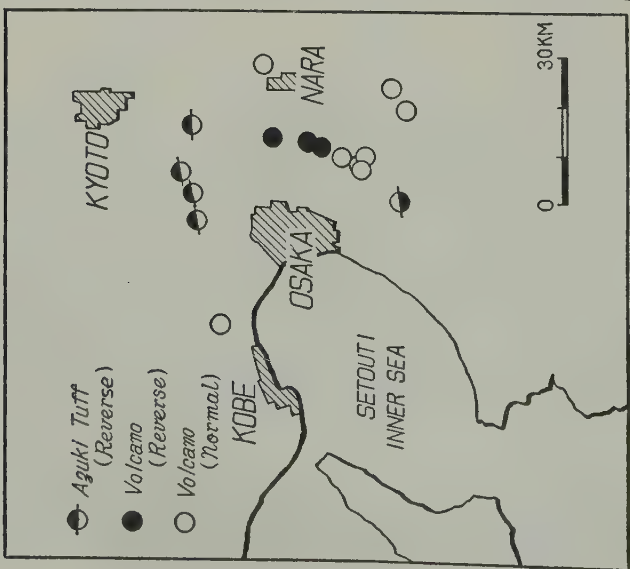
Azuki Tuff (Reverse)
Volcano (Reverse)
Volcano (Normal)
KYOTO
NARA
OSAKA
KOBE
SETOUTI INNER SEA
0
30KM

Fig 5

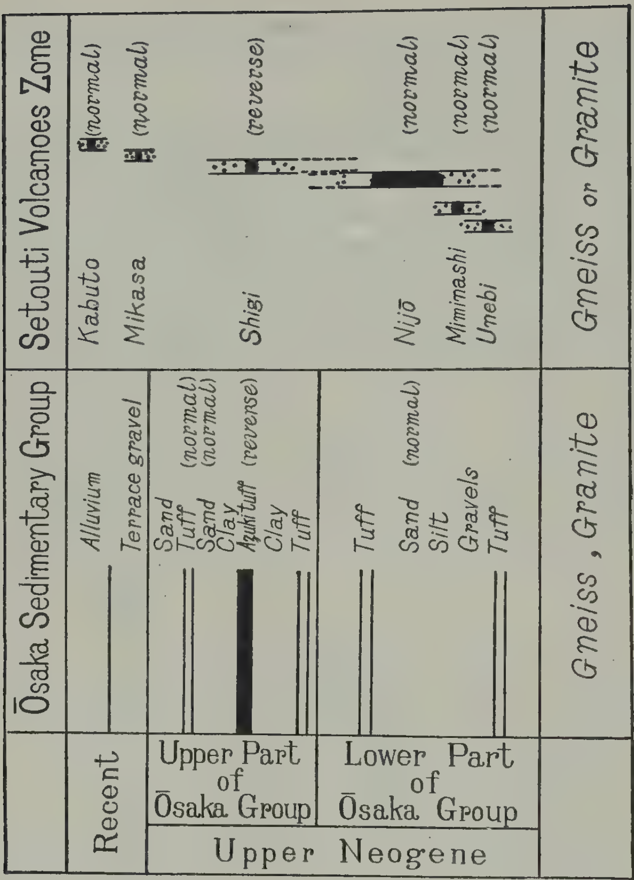
Ōsaka Sedimentary Group
Setouti Volcanoes Zone
Recent
Alluvium
Terrace gravel
Kabuto (normal)
Mikasa (normal)
Upper Part of Ōsaka Group
Sand
Tuff (normal)
Sand (normal)
Clay
Azukituff (reverse)
Clay
Tuff
Shigi (reverse)
Lower Part of Ōsaka Group
Tuff
Sand (normal)
Silt
Gravels
Tuff
Nijō (normal)
Mimimashi (normal)
Unebi (normal)
Upper Neogene
Gneiss , Granite
Gneiss or Granite

Fig. 6

The conclusion of the present study of magnetic polarization of sedimentary and volcanic rocks in Kinki district is schematically shown in Figure 6. It shows that the direction of their magnetization is normal in most cases, but definitely reversed in Azuki tuff and a few volcanic lavas. It may be worth while to note here that, according to Japanese data, the reverse polarization can be found in other volcanic rocks or pyroclastic sediments, and that, as Nagata [8] has already pointed out, they are limited for the cases of Tertiary rocks.

The data of magnetic polarization of both sedimentary and igneous rocks in various localities over the world [5,6,7] have already been accumulated. It may be desirable now to compare those data with each other under some standardization.

In conclusion, the writer wishes to acknowledge his indebtedness to Prof. N. Kumagai for his helpful suggestions as well as for his kind direction throughout the present experiment, and to express his cordial thanks to Mr. J. Takenaka for his help in this study. The author's heartiest thanks are also due to Dr. T. Nagata, of Tokyo University, who gave many suggestions and advice to the present study and kindly read through this article before publication.

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THE 1950 WORLD ISOGONIC CHART

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ABSTRACT

Under a working agreement with the United States Navy Hydrographic Office, a 1950.0 edition of the isogonic chart of the world has been compiled by the United States Coast and Geodetic Survey. Data were collected from all available sources throughout the world. These data were punched into cards, reduced to epoch 1950.0, sorted and averaged by one-degree quadrangles, using machine processes. The average values so derived were entered on base maps and used to control the drawing of isogonic lines. The method has resulted in a saving of many man-years of hand work. The results have been published on one mercator and two polar charts by the Hydrographic Office.

Introduction—The Geomagnetism Branch of the United States Coast and Geodetic Survey, confronted with the job of compiling a new 1950 edition of the isogonic chart of the world, was forced by the shortage of personnel and of working time to develop an entirely new process for the work. The process used is simple, has no unnecessary elaboration, can be clearly described, and permits the performance of a great mass of simple computations by automatic machines. The entire process, being completely new, is here described for the general information of those interested in magnetic cartography.

This isogonic chart was compiled on behalf of the Hydrographic Office of the United States Navy under a working agreement whereby magnetic chart compilation is centralized in the Coast and Geodetic Survey (C&GS). The antecedent chart, dated 1945, was compiled by the Department of Terrestrial Magnetism, Carnegie Institution of Washington, by the use, however, of different and more laborious methods. The C&GS is now designated by statute to be the central agency of the United States for the collection and maintenance of world-wide geomagnetic data. This provision is designed to provide economies and suitable interagency coordination in this field.

Isomagnetic charts portray the magnetic field at a stated instant in a simple way by the use of loci or isolines. One set of lines suffices to show one component of the field. To extend the usefulness of the charts, isoporic lines, or loci of points having equal rates of change, are usually drawn with the isomagnetic lines. Complete definition of the field requires information regarding at least three components. The most

commonly used is the angle defining the direction of the horizontal component. This is called "declination," or, by navigators, "compass variation." Charts presenting this information are widely used in navigation and for other purposes. The 1950 edition of the world isogonic chart was constructed to suit such uses.

The problem—A simple analogy to an isomagnetic chart is a contour chart wherein elevation of the land is represented by lines of equal elevation, with, however, an important difference. Whereas lines of equal elevation on a contour map represent fixed features, the lines on the isomagnetic chart provide an instantaneous view of a feature in motion. A great complexity of magnetic cartography, since it is usually impracticable to make an instantaneous survey of the area to be charted, results from the fact that observations scattered in time as well as in area must first be reduced to the intended epoch before the drafting of the isomagnetic lines can proceed. The secular or long-period variation in geomagnetism varies in regional patterns but according to no discoverable law, so that a complete analysis of the rates of change, as regards both area and time, must be available in advance of the work of reduction. Since any station by reason of its location has a unique reduction value depending on both location and date of observation, and, since for the world approximately 70,000 stations must be considered, it is seen that the problem is one of great magnitude and complexity.

In view of the large physical task of reducing the great number of station values, many expedients have been devised in the past. One of the principal methods of attack has been to base a new compilation upon the last previous drawing, taking account of new additional station values available in the meantime, then moving the lines to new positions in accordance with available information regarding secular change. This is determined through study of magnetic observatory results and of repeat observations made at certain selected field stations. This method would be reasonably satisfactory if the general magnetic pattern were essentially smooth and undistorted. This, however, is not the case. On the contrary, the pattern has great complexity, with almost infinite varieties of minute configurations. These irregularities, commonly called anomalies, are due to magnetic conditions in localities and remain therefore geographically fixed. Any practical method of recompilation based on the idea of "moving over" lines from a preceding isomagnetic chart develops errors and inconsistencies, because such methods tend to change the shapes or positions of the anomalous configurations. While these are not even approximately known in some areas, and in any case would not, for practical reasons, be completely charted, nevertheless the general chart under consideration was intended to show some principal anomalies. The method of moving over lines was therefore unsatisfactory.

A better method, available in the United States where the density of observations is sufficiently great, involves separation of the field into two component parts, each of which is separately compiled on a construction chart. One, the so-called datum chart, is highly smoothed to eliminate local anomalies, and represents the regional average values with generally systematic gradients. The other of these charts shows the remainder of the field, freed of the regional values, thus constituting an anomaly chart specifically showing sub-regional departures from normal. The former, or datum chart, is subject to secular change and can be recompiled for

any date. The anomaly chart, once compiled in sufficient detail, is fixed. A recombination gives the desired isomagnetic chart for the desired date. This process of compiling anomaly charts involves laborious operations; moreover, the amount of data generally available does not favor such treatment uniformly over the world. The method was, therefore, considered undesirable for the project under discussion.

Major jobs of isomagnetic chart compilation in the past have tended, because of the complexity of the operation, to become very largely the personal and highly individualistic work of single magnetic specialists, a practice limiting the possibility of cooperation of a group of workers, and generally contributing in the past to very late issue of charts.

The 1950 process—The problem obviously was to reduce the processing method to a simple and easily described and understood operation, permitting joint work by a number of people, and as far as possible to reduce the quantity of routine computational work to be performed by hand. This indicated recourse to automatic computing machines, using punched cards, which are capable of a remarkable variety of computational steps and tabulation of results. The following unique method permitted the successful completion of the 1950 chart compilation project on schedule.

Following an initial decision to employ magnetic station declination values originating only within the past 50 years, all available values from world sources were collected—a total of 78,000 declination observations at more than 70,000 different stations. This collection unfortunately does not contain data later than about 1946 from most Soviet controlled areas. A systematic arrangement of these data was prepared by key-punching the essential details on cards, which thereafter constituted a convenient file of material readily sorted by machine according to location, date, or any desired scheme.

Time-declination curves, constructed for all observatories operating during this time, as well as for some 145 selected repeat stations having adequate data, showed the declination at any time, and by their slopes, the rate of secular change of declination. Such curves, exhibited a general tendency to have changes in the slopes of the curves at similar times. These changes of rate were termed "impulses." Eleven dates were selected to correspond with general changes of rate or "impulses." These are 1908.0, 1910.5, 1913.8, 1917.0, 1920.0, 1924.0, 1928.0, 1933.5, 1937.0, 1940.5, and 1944.5.

The compiled "impulses" were in sufficient distribution to control the drawing of world "iso-impulse" charts, one for each "impulse" epoch. This facilitated the quick graphical determination of "impulse" at any place for any epoch.

For each of about 1,150 available repeat stations, the time and declination values of the two best observations were listed, as well as the "impulses" at each epoch, scaled from the "iso-impulse" charts. This information permitted a quick and simple hand computation, resulting in the rate of annual change for each period between epochs for each of these stations.

A world map of curves of equal annual change for each period was drawn, to accord with a position plot of the annual change values so derived for all observatories and repeat stations. From these maps the annual change rates, for each period, were scaled and tabulated for all intersections of a world grid representing $10\frac{1}{2}°$,

$20\frac{1}{2}°$, $30\frac{1}{2}°$, etc., in both latitude and longitude. These grid-point values were punched into master cards used to control subsequent machine operation. This selection of points facilitated machine interpolation of values for the centers of one-degree quad areas.

The processing from this point to obtain 1950.0 reduced declination values of all stations of record since 1900 is a matter of simple arithmetic, but with individual operations numbered in the millions. Here is the province of the automatic computer, which disposed of the large numbers of repetitive computations in extremely brief periods of time. The actual steps are not enumerated here, since they are of only incidental interest, but it is noted in passing that this computational facility saved several man-years of hand work and can be credited with the completion of the job on time.

In brief, this is what was accomplished with no hand work except that of machine operators:

The station cards were sorted by 10-degree quads, annual change rates for each of the applicable periods between impulses interpolated from the master-card values, the total change (rate multiplied by time) computed for each period, these total changes per period summed and the resulting total reduction applied to each station value. Within each 1-degree area all reduced station values were grouped, and mean declination, latitude, and longitude derived and automatically tabulated. The foregoing involved various "runs" of the cards and a variety of operations, during which checks of various kinds were possible, with occasional limited re-runs and rejection of dubious values.

The final preparation of the chart involved the plotting of mean declination values for 1950.0 at mean position within each one-degree quad, the drawing, justifying, and smoothing of the isogonic lines to suit the plotted values, and the addition of the isoporic, or equal annual change, lines for 1950.0 previously prepared as one of the construction steps. The results have been issued on one mercator and two polar charts by the Hydrographic Office, United States Navy, which has primary responsibility for publication of world magnetic charts.

SOUTHERN EXTENT OF AURORA BOREALIS IN NORTH AMERICA

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ABSTRACT

Results are presented for the first 11 years of a study of the frequency of overhead auroras in North America as a function of latitude in a region south of the auroral zone. The data have been averaged in various ways so that monthly and annual variations are demonstrated. It appears that there is a relatively constant level of auroral activity throughout the year in the region 58° to 60° geomagnetic latitude, while auroras appearing overhead south of these latitudes are more frequent during equinoctial periods. Auroras have been seen as far south as 52° during this period every month of the year. Correlation of auroral frequency with sunspot number is not high on a month-by-month or three-month running-mean basis.

Introduction

Since 1938 a study of the aurora has been carried on at Cornell University under the sponsorship of the National Geographic Society. In the last 18 months, additional support has been given by the United States Army Signal Corps. From the beginning, one phase of this work has been the collection of reports of visible auroras from a large number of volunteer observers scattered throughout the northern part of the United States and the southern part of Canada. It has been possible to compile the data so obtained in such a way that the southernmost latitude at which aurora was seen on any given night is known. These data on southern extent are summarized here.

Apparently this is the first time that detailed reports of aurora from such a large area of North America have been compiled over a period of this duration. These data are now available for slightly more than a complete 11-year sunspot cycle. This part of North America has the advantage for such a study that the geomagnetic latitude exceeds the geographic latitude by the maximum amount. The result is that more hours of darkness are available for summer observations at a given geomagnetic latitude than anywhere else in the northern hemisphere. Early work of this project was reported some time ago [see 1 and 2 of "References" at end of paper]. By 1945 the reporting of auroras had crystallized into essentially the form used today [3].

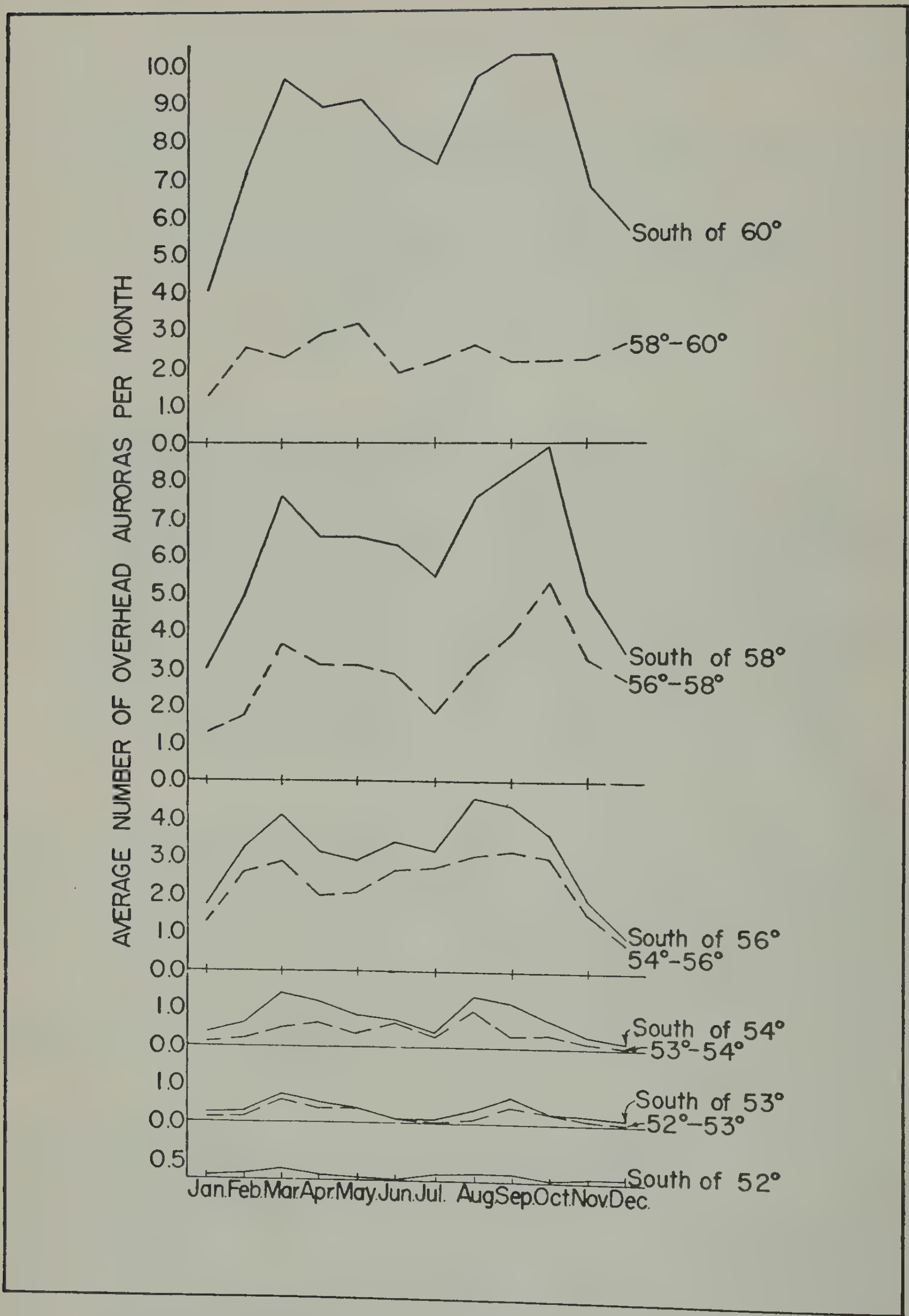

FIG. 1—AVERAGE NUMBER OF OVERHEAD AURORAS PER MONTH DURING PERIOD 1939-1950

Index of aurora size

There have been many attempts to establish an index of aurora size. One such index is based on brightness estimates. Another is based on the latitudes at which auroras could be seen somewhere in the sky. Still others are more complicated, and take into account the various forms of aurora, the activity, and duration, as well as the above factors [4].

Although brightness might be a good index of aurora size if it could be measured quantitatively, any measure of brightness must be a function not only of the brightness of the aurora but also of the amount of moonlight and cloud cover. Individual judgment also enters into such an index to an undesirable extent. Some complicated index might be desirable, but it is fairly difficult to determine exactly how one should be made, and in any event evaluation of the data would probably become tedious.

The figure used here is the southernmost geomagnetic latitude at which the aurora could be seen overhead. While this is not necessarily the best index of auroral size, it is a fairly convenient one to determine from such data as were available for this analysis. It can be established more accurately than a brightness index or one based on the statement that an aurora was seen somewhere in the sky at a particular latitude. Southern extent can also be used to show the way in which the auroral zones expand during the equinoctial periods.

The number of observers reporting a given aurora is not large enough for it to be certain that some observer will be at the point of southernmost overhead extent, but it is possible to compute the southern extent from the elevation above the horizon and direction in which a display is seen (see Appendix). This has been done on the assumption that the height of the lower border of most forms of aurora is 100 km [5, 6]. Corrections have been made for auroral forms which are usually measured at greater heights than this.

Monthly frequency of auroras

The data on southern extent have been compiled for the period of January 1939 through July 1950. It should be noted that these data cover a longitude range of about 45° (from Maine to Montana), with the greatest concentration being in the eastern 20° of this range. Because of the large area included in the observations, we believe few auroras overhead south of 60° have not been reported. The number of auroras reported overhead in zones whose width is two degrees of geomagnetic latitude has been obtained for each month of this period.

In Figure 1 the average number of auroras for a given month is plotted. Curves are given for the number of auroras seen overhead within the various zones and for the total number of auroras south of specified geomagnetic latitudes. In terms of geographic latitudes in the eastern United States, the zone "south of 52°" means south of New York City; 53° is approximately the southern boundary of western New York State; 54° corresponds roughly to the latitude of Buffalo; 56° geomagnetic latitude is just south of Montreal; 58° crosses the northern tip of Maine; 60° is north of any major city but south of James Bay. An aurora which is overhead in the 58° to 60° zone is just barely visible above the northern horizon in Ithaca, so

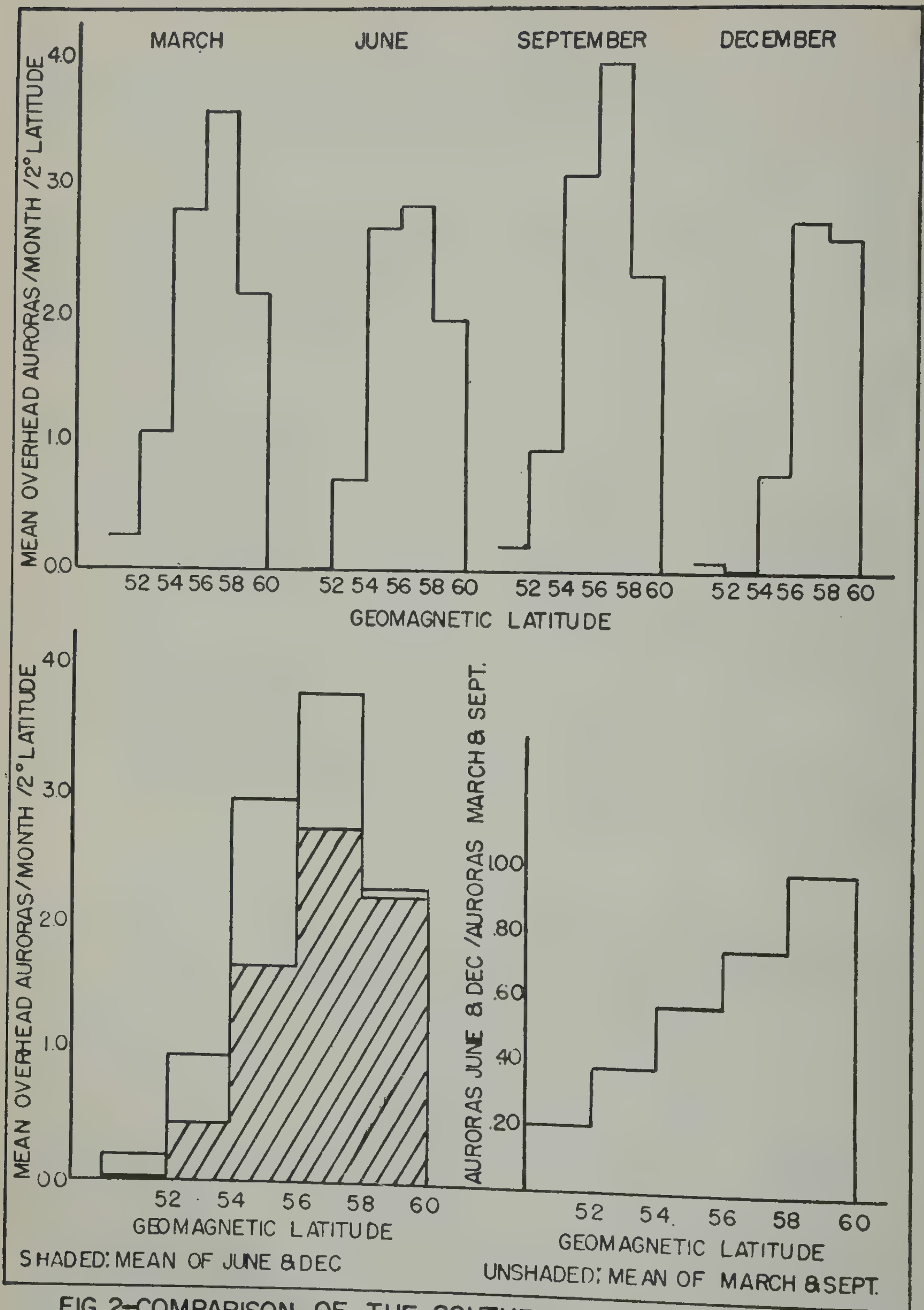

FIG. 2—COMPARISON OF THE SOUTHERN EXTENSION OF THE AURORAL ZONE IN EQUINOCTIAL MONTHS WITH THAT IN SOLSTITIAL MONTHS

that any aurora included in this study would be visible in Ithaca on a good observing night.

Figure 1 shows the well-known peaks of auroral activity near the equinoxes. However, in the 11.5-year period studied, at least one aurora of the largest size was reported during each month of the year except May, June, and October. Auroras south of 53° occurred at least once during this interval for every month. It is not known whether the shift of the months of peak activity for the various zones is real or merely due to the relative paucity of observations of auroras of the larger size. While the midsummer solstitial valley in the curves is not as deep as that in midwinter, it is not known whether this is a true phenomenon or whether it is due to the fact that simultaneous cloudiness may occur frequently enough during the winter months to prejudice the data. It seems likely that auroras really are less frequent in midwinter.

It should be noted that the number of auroras seen in the northernmost zone is subject to relatively little variation throughout the year. This is seen more clearly by examination of Figure 2, in which the months of the solstice are compared with the months of the equinox. The average number of auroras seen overhead throughout the period of the study in the indicated months is plotted in the upper histogram as a function of the geomagnetic latitude. The apparent decline as we go from the 56-58° zone to the 58-60° zone is probably not real, as the possibility of missing an aurora overhead in the latter zone is fairly great because it is well north of nearly all the observers. It is unlikely that many auroras overhead in the 56-58° zone were missed, but it is quite possible that many were missed in the 58-60° zone. In spite of this, it seems reasonable that the chance of missing these northernmost auroras investigated is as likely in one month as in another, so that ratios of the number of auroras overhead at this latitude in the solstitial months to those in the equinoctial months should be reasonably accurate. The means for the two pairs of months are given in the lower left-hand histogram and the ratio of solstitial auroras to equinoctial auroras is given in the lower right-hand histogram. The decrease in the number of auroras extending far to the south during the solstitial months is evident.

The implication of this is that the auroral activity north of 58° (geomagnetic) is relatively consistent throughout the year, but the auroral zone has greater tendency to expand during the equinoctial periods. Examination of Figure 1 demonstrates that this is merely a tendency and the expansion may occur at least occasionally in every month of the year.

Annual frequency of auroras

The data on southern extent have been averaged on an annual basis and the results are plotted in Figure 3. It will be noted that the number of most southerly auroras shows a fairly definite tendency toward a minimum in 1944 at the minimum of the sunspot cycle, but that the number of smaller ones does not follow the curve of sunspot number nearly so well.

It will be noted that more than ten auroras were seen south of 54° (overhead in Ithaca or farther south) during the years 1939, 1940, 1947, 1948, and 1949, and at least two were seen overhead this far south even in the sunspot minimum year of 1944. Only in one year (1944) were there less than 25 overhead auroras south of 56°

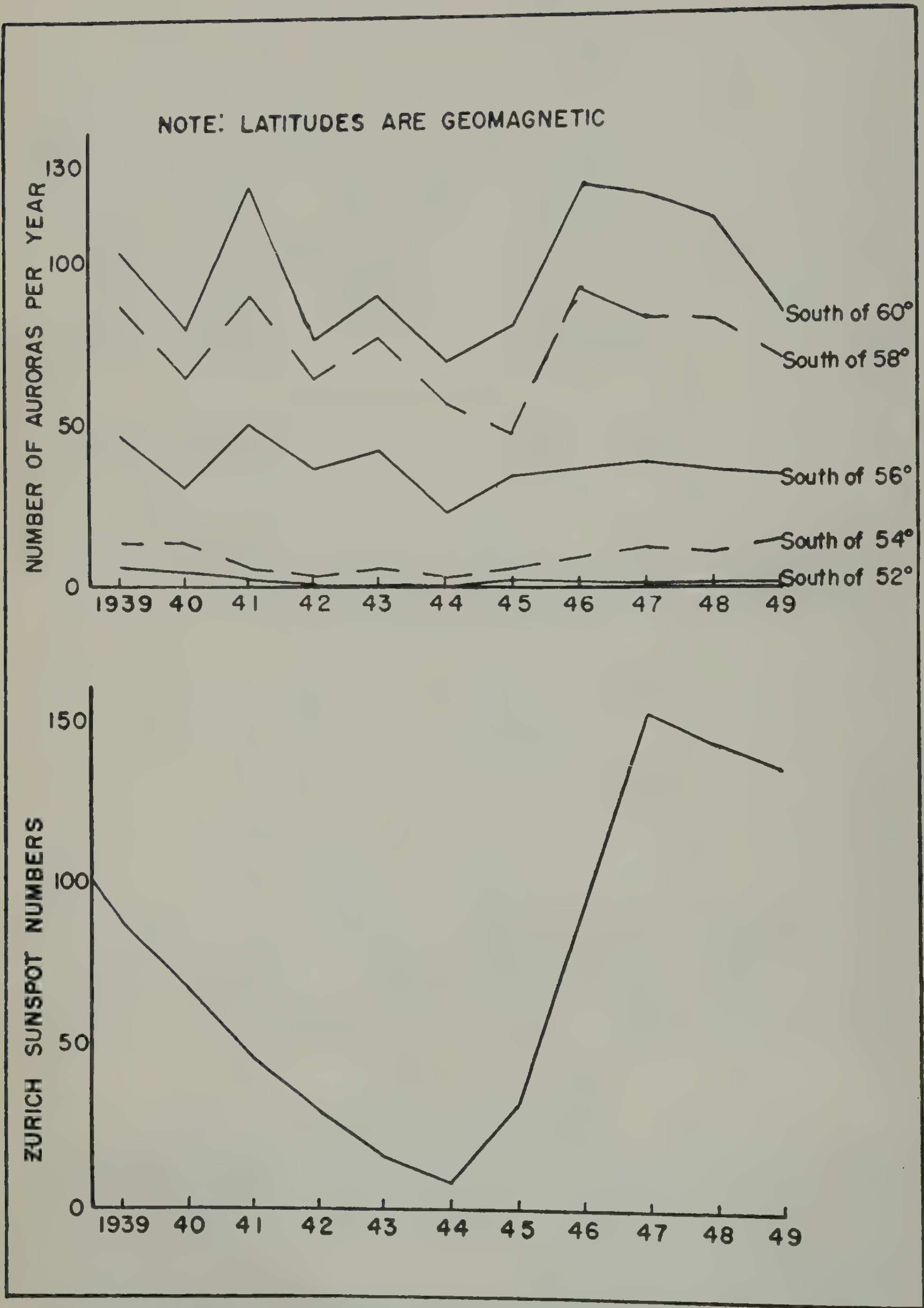

FIG. 3—NUMBER OF OVERHEAD AURORAS PER YEAR (1939-1949) AND MEAN ANNUAL ZURICH SUNSPOT NUMBERS

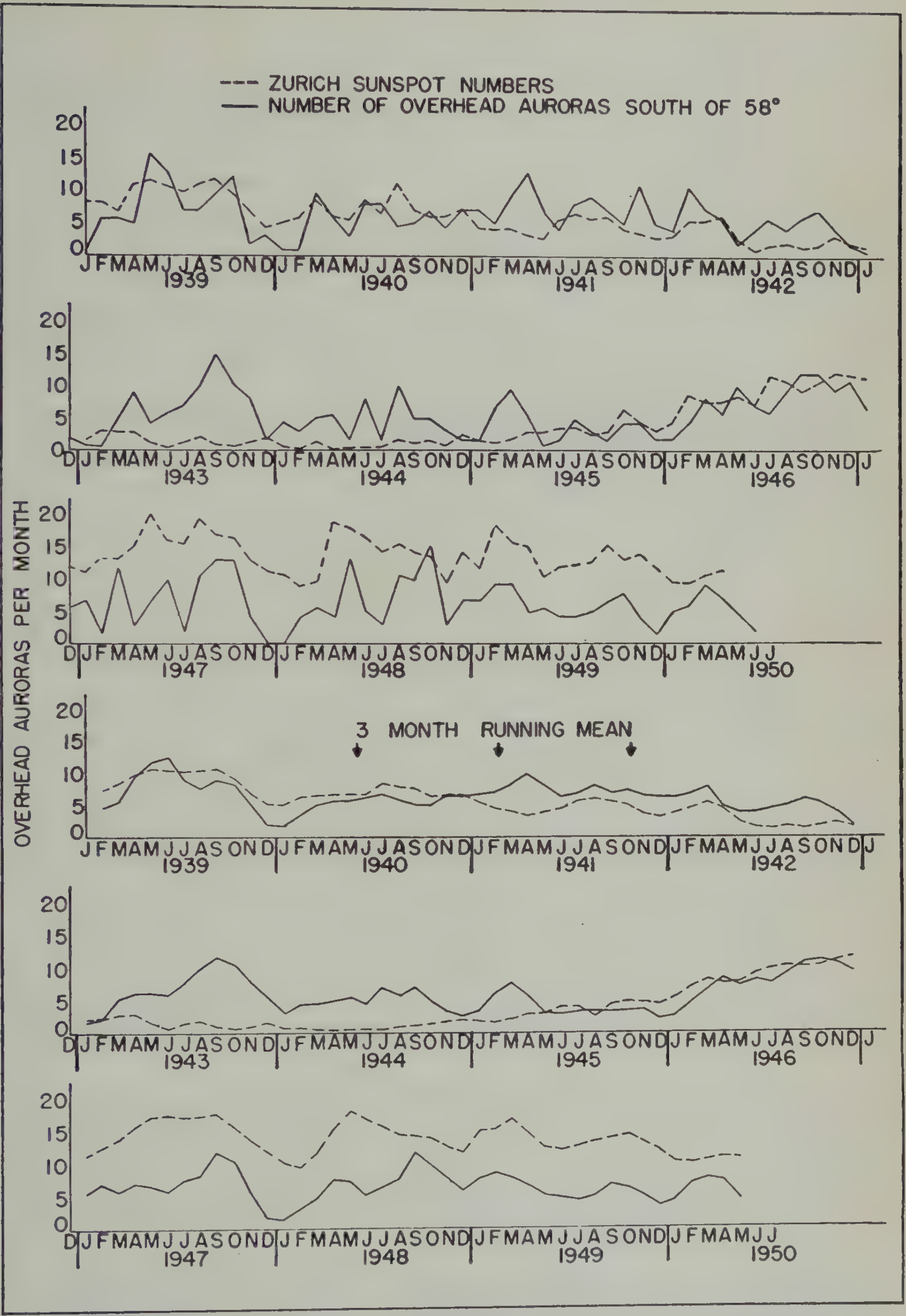

FIG.4–NUMBER OF OVERHEAD AURORAS SOUTH OF 58° GEOMAGNETIC LATITUDE COMPARED WITH ZURICH SUNSPOT NUMBERS

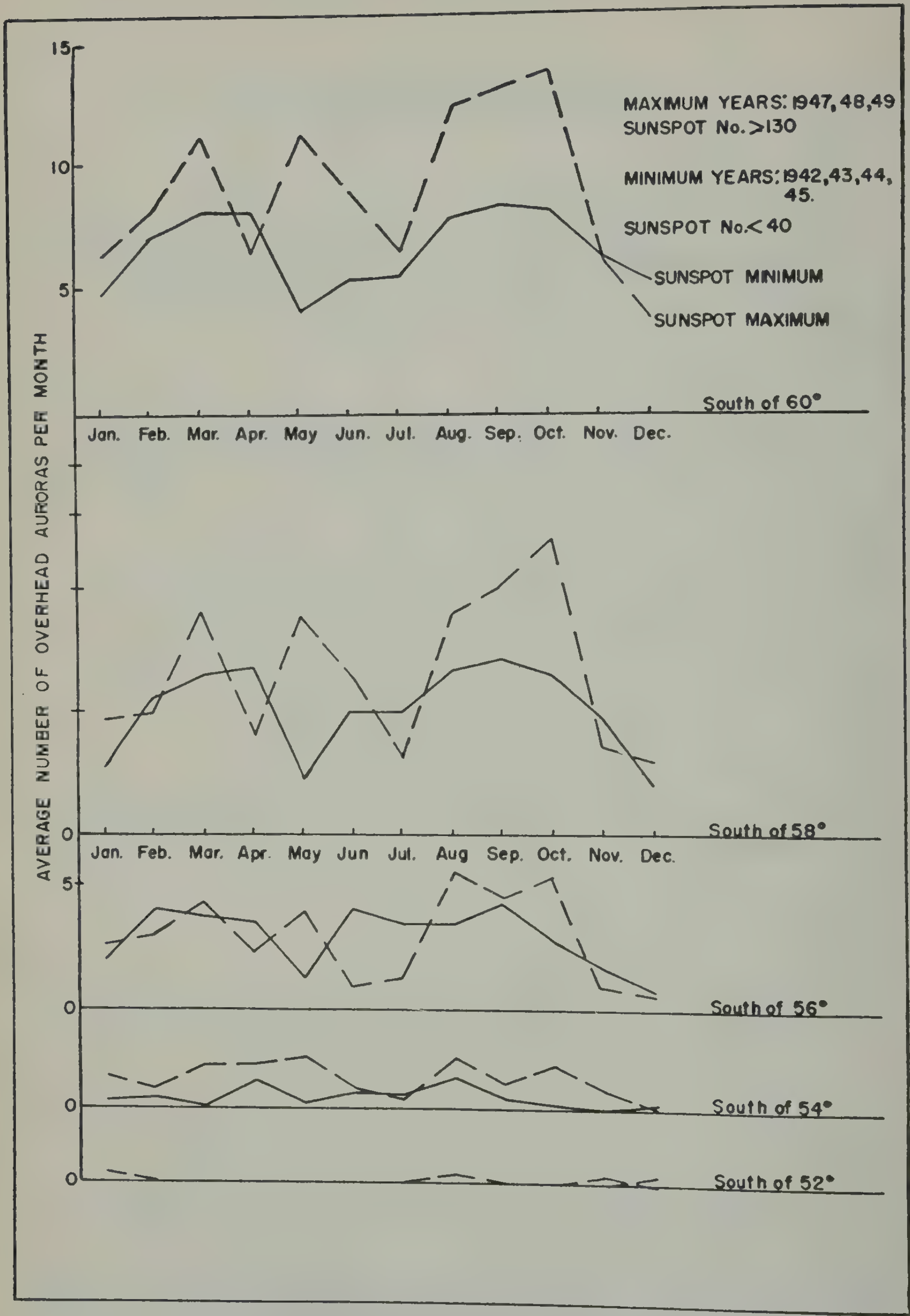

FIG. 5—AVERAGE NUMBER OF OVERHEAD AURORAS PER MONTH IN YEARS OF SUNSPOT MAXIMUM AND MINIMUM

(Montreal), and in 1941 there were 50 seen overhead at these latitudes. Since any aurora in this study could be seen from Ithaca under proper conditions, we conclude that at least 70 auroras were visible from our latitude (neglecting clouds) in every year of the study, and as many as 125 appeared in one year.

Correlation with sunspots

It is well known that individual large auroras and magnetic disturbances are frequently associated with particular sunspot groups. An attempt to correlate the number of auroras seen with the Zurich sunspot numbers was not very successful. The annual means of the sunspot numbers are shown in Figure 3 beneath the annual means of auroral number. While the sunspot curve is quite smooth, showing a very pronounced valley and peak, the auroral curves are quite irregular. The peak of the sunspot cycle does coincide in a general way with a peak of auroral activity.

Since the annual means did not show very good correlation, it was decided to examine the situation with regard to monthly figures. The monthly figures of numbers of aurora seen overhead south of 58° geomagnetic latitude are plotted throughout the period of the study in Figure 4 along with the monthly mean Zurich sunspot numbers. It can be seen that little correlation exists. A correllogram was started, but the results for the first three years showed so little correlation that it was not continued.

It was thought that three-month running means of the auroral and sunspot data might correlate somewhat better, and consequently the lower curve of Figure 4 was constructed. Correlation between the running means is better than that of the non-averaged data, particularly during the periods of high sunspot numbers. During the period of sunspot minimum, however, no correlation is evident. It did seem, therefore, that correlation of the running means of sunspot numbers and auroral numbers does exist in the periods when sunspot activity is at a peak, but that no correlation can be found, at least for auroras south of 58° geomagnetic latitude, during periods of low sunspot activity.

A comparison of the monthly frequency of auroras during the years of sunspot maximum with that during the years of sunspot minimum is shown in Figure 5. It can be seen that the greatest increase during the sunspot maximum years occurs during the equinoctial periods, but the differences between the two sets of data are not clear enough to permit drawing any real conclusions.

Summary

Results of the first 11 years of a study of the frequency of overhead auroras in North America at different geomagnetic latitudes south of the auroral zone have been presented. The data have been averaged in various ways so that monthly and annual trends are demonstrated. It appears that there is a relatively consistent level of auroral activity throughout the year in the vicinity of 58° to 60° geomagnetic latitude, and that auroras seen overhead south of this latitude are more frequent near the time of the equinoxes. Nevertheless, auroras have been seen overhead at all times of the year at points as far south as 52° geomagnetic latitude. Correlation of aurora frequency with sunspot number is not high.

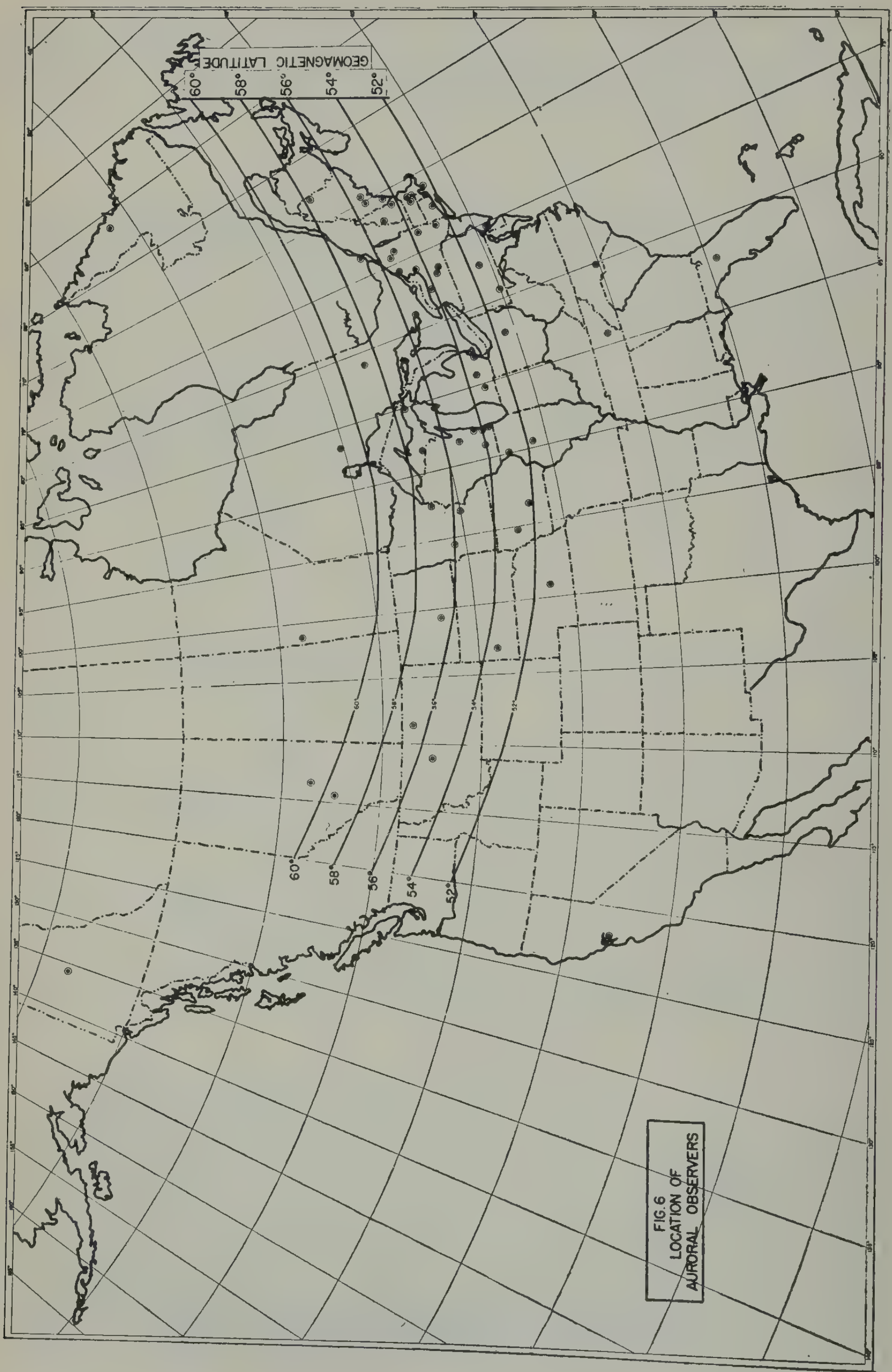

FIG. 6 LOCATION OF AURORAL OBSERVERS

APPENDIX

Compilation of southern extent data

The volunteer observers providing data under the National Geographic Society-Cornell University aurora project are spread throughout the northern part of the United States from Maine to Montana and across southern Canada in roughly the same longitudinal range. During some of the largest auroras as many as 50 different persons have reported. On the other hand, many of the smaller auroras have been reported by only one observer. This is particularly true during the winter months when cloudy skies are common throughout most of the area. A map showing the location of many of the observers who have participated is shown in Figure 6.

Many of these observers are amateur astronomers and cooperation of various organizations of amateur astronomers has been quite helfpul. The American Association of Variable Star Observers, the Royal Astronomical Society of Canada, and the Milwaukee Astronomical Society have made a special effort to maintain continuity of observation. Reports from certain stations of the United States Weather Bureau have been helpful, particularly for the hours after midnight. Other observers have become interested after reading the article in the National Geographical Magazine [7]. Visual observations are carried out nightly at Ithaca, and photographs are made of significant features of the auroras.

Observers use two types of report form. One is a monthly summary, listing the hours during the month when observations were made and indicating the cloudiness of the sky and brightness of aurora when present. Reports are also received for individual displays giving more detailed data on brightness, elevation, direction, color, and auroral forms seen at the various times. Many of these reports are very complete, giving minute-by-minute accounts of the aurora.

The southern extent for each observation is computed from the elevation data and location of the observer. It is assumed that arc forms are at a height of 100 km and ray forms somewhat higher. It is then possible to assign coordinates to the point at which the aurora was overhead. No attempt at evaluation of the southern extent of glow is made. After all computations have been made for a given night, the reports from the various observers are grouped and a maximum southern extent figure is assigned to each three-hour interval. For the data prior to 1947, this and a great deal of other information have been summarized on punched cards and the cards have been used with automatic business machines to print the data in the form of tables. These cards may also be used for various types of statistical analysis.

The southern extent data for the various three-hour intervals during a given night are surveyed, and the southernmost extent of the aurora during the night is the figure assigned to that particular date. It should be pointed out that data for the time prior to midnight are likely to be more reliable than those for later times, because the number of observers active before midnight is much greater than after midnight.

The above analysis was carried out using the daily southern extent figures thus made available.

Acknowledgments

The authors wish to acknowledge the helpful suggestions and encouragement of Dr. H. G. Booker. The Signal Corps sponsorship was under Contract W-36-039-sc-44518 with Signal Corps Engineering Laboratories.

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[4] See, for example, B. W. Currie and C. K. Jones, Terr. Mag., **46**, 269-278 (1941); and I. L. Thomsen, Terr. Mag., **52**, 453-467 (1947).

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A V.H.F. PROPAGATION PHENOMENON ASSOCIATED WITH AURORA

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ABSTRACT

Anomalous propagation observed by radio amateurs at frequencies of 28 to 148 Mc during displays of aurora polaris is described. This phenomenon, first correlated with aurora in 1939, is characterized by the following features:

(1) A very high fading rate, such that voice modulation is rendered unintelligible
(2) Directional antennas give best results when pointed north (toward the aurora)
(3) Lack of skip effect
(4) Little change in polarization

Comparison of reports of the radio phenomenon with visual observations of aurora indicates this effect is most common with auroral displays extending below 56° geomagnetic latitude, although the lack of amateur stations at high latitudes may influence these data. Until more precise fading data are available, development of a suitable theory is deferred.

Introduction

For a number of years, amateur radio operators have observed certain anomalous propagation conditions presumably connected with displays of aurora polaris. These conditions take the form of propagation to distances not likely to be achieved by other means in the amateur bands from 28 to 148 Mc. Normally this propagation is characterized by an extremely rapid fade or modulation. Best results are usually obtained with directional antennas pointed north, regardless of the direction of the line connecting the two stations.

Evidently this effect was first noted on the 56-Mc band during the 30's. Apparently the first reference to it appears in *QST*, May 1939 issue [see 1 of "References" at end of paper]. During 1939, a column entitled "On the Ultra Highs" became a regular feature of *QST*, and subsequent reports prior to the war were in this column. After the war, this column was given the name, "The World Above 50 Mc" and post-war observations of this phenomenon have been reported under this heading. Reports of "auroral propagation" have also appeared in *Radio* (pre-war) and *CQ* (post-war), in the columns entitled "U.H.F." and "V.H.F.-U.H.F."

After the first observation was reported in 1939, reports of 56-Mc "aurora effects" became commonplace. By 1941, references to them in the above-mentioned columns were in the off-hand manner of references to something known to everyone. After the war, the effect of aurora was noted on the new 50-Mc band. In June, 1948, apparent observations of this type of propagation were made in the 144 Mc band. In the March 1949 issue of *QST*, it was recorded that two-way communication "via the aurora" had been made on 144 Mc during the evening of January 24, 1949. Since that time many additional observations of such effects at 144 Mc have been noted. "Auroral propagation" of 40-Mc frequency modulation broadcast stations was reported in 1941 [2].

The first published mention of the "aurora effect" in *QST* gives a colorful description of it, repeated many times since [1]:

"W2AMJ makes some interesting observations regarding conditions noted on 56 Mc: The Northern Lights were going to town on the night of February 24th. I got on 56 Mc at about 8:20 p.m. I sent a CQ on i.c.w. and was rewarded with a call from W8VO in Akron, Ohio, also on i.c.w. His carrier was S9 but had the most peculiar sound to it, with or without modulation. He told me that the band had been open out there for about an hour. He worked W2KLZ earlier in the evening. I swung over to 'phone, and he advised me that the 'phone was unintelligible. Then he shifted to 'phone, and boy, if you ever heard inverted speech you should have heard his. It sounded nothing like voice modulation. We continued the contact on i.c.w., S9 both ways. At no time was there any appreciable fading. The signal was just like it was around the corner, except for the peculiar combination of howl and roar that accompanied his carrier. We signed about 8:45, and a little after 9:00 W8VO dropped out of the picture just like somebody cut the wires with a pair of pliers. W1EYM called me then. I did not notice anything unusual on his signal, but he told me about the Northern Lights and for the past hour had noticed the characteristic rumble on quite a few 2nd District signals. We also found that practically all of the 28- and 14-Mc harmonics had the same characteristics that W8VO's signal had. W8AGU, Penfield, N.Y., heard both W8VO and myself; he also noticed the strange condition, which he called exceptional *QSB*."

During and since the war, a number of observations of radar echoes, apparently from auroral displays, have been noted [3]. These observations were made with radar and meteor-study equipment operating in the region of 30 to 100 Mc. Apparently these echoes are manifestations of the same ionospheric phenomenon which causes the anomalous propagation reported by the amateurs.

Characteristics of auroral propagation

The phenomenon known by the radio amateurs as "auroral propagation" has certain very definite characteristics which distinguish it from other modes of propagation possible at the frequencies involved.

(1) There is at all times a very high fading rate. No measurements have been made to determine this rate, but some idea of it may be obtained by examining the various statements made by persons who have observed this effect. The fading is normally so rapid that modulation is rendered unintelligible, and high speed telegraphy is impossible because of a blending of the characters. One statement fre-

quently encountered is that a signal so propagated sounds the same whether the beat frequency oscillator of the receiver is on or off. Estimates of the fading rate range from as low as 30 cps to as high as 15 kcps. One was that "it sounds like the tearing of a sheet, whatever frequency that may be." At any rate, it appears that the fading must have components up to at least a few hundred cycles per second.

Apparently the fading rate increases with frequency (as would be expected if it is caused by doppler shift). Reports from the 28-Mc amateur band indicate that voice modulation is frequently intelligible on that band during "auroral propagation" and it is stated that, during the largest auroras, voice modulation may be intelligible on the 50-Mc band. On the 144-Mc band, however, it is never possible to communicate other than by telegraphy.

(2) Another distinguishing feature of this type of propagation is the fact that signals are strongest when directional antennas used at both stations are pointed to the north, regardless of the actual geographical direction between the stations; although occasionally some other direction not far from north is preferred. Many times the "auroral flutter" may be heard, if the antennas are pointed to the north, on a signal receivable over a direct path. Presumably this "flutter" may be heard as an indirect component of signals from near-by stations, even at times when no signals propagated "entirely by aurora" are audible.

When there is an aurora overhead, the optimum direction is not so well defined. In fact, signals of the same intensity from a given station may be heard regardless of the azimuthal direction in which the receiving beam antenna is pointed. This would seem to indicate that these signals are propagated by some mechanism in which scattering takes place in the region in which auroral light is generated.

(3) This phenomenon may be distinguished from sporadic E propagation by the lack of any skip effect. At these frequencies, signals propagated via sporadic E are seldom heard at distances shorter than 600 miles. "Aurora propagated" signals may be heard at any particular time coming from stations whose distances from the receiver range from a few miles to as high as 500 miles. Recently there have been some reports of "auroral propagation" from distances as high as 900 miles.

Stations making contact by this means may be at approximately the same longitude but different latitudes, or they may be at the same latitude and different longitudes. Apparently the fading rate is somewhat higher if the stations are in a north-south line.

(4) It appears that the polarization of the received signal is essentially the same as that of the transmitted signal, but information concerning this is very meager.

Originally this phenomenon was reported by those who observed a coincidence between visible auroras and "fluttery" signals. It appeared that the best results, from the amateurs' standpoint, were obtained with some fairly static auroral form in the north. A corollary of this is that "auroral propagation" was observed occasionally at the beginning and end of a large auroral display and was not present when the aurora was overhead.

Soon after this phenomenon became recognized, it was observed that such signals could be heard at times when no aurora could be seen. Frequently the stations observing these signals were located in regions where the sky was cloudy. The very nature of amateur radio operation precludes visual observations of the aurora for

many amateurs even when the sky is not overcast. In fact, many of the stations reporting "auroral propagation" are in cities where the artificial illumination level is so high that auroras are normally invisible.

Signals of this type also have appeared many times during day or twilight when it would have been impossible to see an auroral display. Perhaps the earliest afternoon report was $14^h\ 00^m$ local time. Occasional reports during the very largest magnetic disturbances have indicated that the phenomenon could continue as late as $10^h\ 00^m$ local time. Reports of this type of propagation for the period between midnight and $06^h\ 00^m$ are scarce because very few amateurs operate during these hours.

Most reports of this type of propagation have come from the United States and Canada, but occasional reports have been received from Europe. VHF propagation of this nature has also been reported during displays of aurora australis in Australia and New Zealand.

During the largest displays, reports have come from points as far south as the coast of the Gulf of Mexico (42° geomagnetic latitude). Most reports, however, are from that portion of the United States and Canada surrounding the Great Lakes, and from New England. This is partly due to the high concentration of amateur stations in this region. Geomagnetic latitudes of most reporting stations lie within the range 50° to 56°. Geomagnetic latitudes of the stations observing auroral propagation in the Southern Hemisphere are not known.

Comparison with aurora data

It was decided to determine whether this phenomenon was, in truth, associated with aurora before instituting a thorough scientific study of it, and if it was, to determine what southern extent the aurora must have for this type of propagation to be reported. There were available the data on southernmost extent of overhead auroras compiled by Gartlein under the National Geographic Society-Cornell University Aurora Project [4]. These data were available for the period 1939 to date and amateur radio reports of auroral propagation cover the same period (with a gap during 1942 to 1945, caused by cessation of amateur radio activities during participation of the United States in World War II).

Reports of auroral propagation observed by amateur radio operators have been obtained from a number of sources. The appropriate columns of the amateur journals, *QST*, *Radio*, and *CQ*, were searched for all such reports, and a list of dates and times was compiled. Additional information has been obtained in correspondence with the VHF editors of these magazines.*

Besides occasional descriptions of the signals propagated in this manner, the reports in the journals usually list the time of contact and the call letters of the stations involved. Frequently, however, the only reference is a mention that "auroral propagation" occurred on some date, with no time or station given. Because of the difficulties of ascertaining the location of the various stations in past years, no use has been made of the call letter data.

The times of contact have been classified into three-hour periods, corresponding

*Mr. E. P. Tilton of *QST* and Mr. E. M. Brown of *CQ*.

to the three-hour periods into which the aurora data is subdivided; that is, periods starting with midnight GMT. Some of the reports compiled refer to contacts made by this mode of propagation and others to observations of the flutter fading, when the antenna was pointed north, on signals propagated through the troposphere. The reports were studied in the hope that some reliability index could be assigned in each case. Such indices were established, ranging from A for certain auroral propagation to D for cases in which the validity of the report is extremely doubtful. In the final analysis, instances labeled A and B (normally referring to cases for which a large number of reports was available) were lumped together, and instances labeled C and D (usually only one or two reports for a given night) were lumped together.

It is likely that such propagation has not been reported on many occasions when it was possible. Before the war the equipment in use by the amateurs at these frequencies was in most cases insensitive, which meant that weak signals were frequently missed. Until recently, most operation on these amateur bands has been by stations employing voice modulation only, and many instances of auroral propagation were undoubtedly missed because of garbling of modulation by the high rate of fading. Because of the relatively limited ranges possible on these bands under normal conditions, amateur activity in them has never been very great, and many instances of auroral propagation may have been missed because there were no stations operating at the right time and place. This situation has been improved recently by operation of certain "beacon stations" near the 50-Mc band by the Canadian Defense Research Board; amateurs tuning to these beacons at any time report unusual reception.

A study of coincidences between reports of auroral propagation and auroras visible overhead at given geomagnetic latitudes has been made. The maximum southern extent of aurora for each day on which auroral propagation was reported has been compiled in Table 1. The ratio between the number of days on which

TABLE 1—*Number of days on which "auroral propagation" was reported compared with maximum southern extent of the aurora, 1939 to 1941 and 1946 to 1950*

Reliability index for radio data	Geomagnetic latitude of maximum southern extent						
	< 52°	52°-53°	53°-54°	54°-56°	56°-58°	58°-60°	> 60° or none
A or *B*	8	12	11	23	11	4	6
C or *D*	1	1	5	12	11	5	20
All	9	13	16	35	22	9	26
	Total days of aurora of indicated maximum southern extent						
	10	24	37	197	278	169	1687

"auroral propagation" occurred during an aurora of a given southern extent and the total number of days on which the aurora had such a southern extent has been converted into a percentage; or, in other words, a figure has been obtained for the percentage of the total number of days of a given auroral southern extent during which auroral propagation was reported. This is shown in Figure 1.

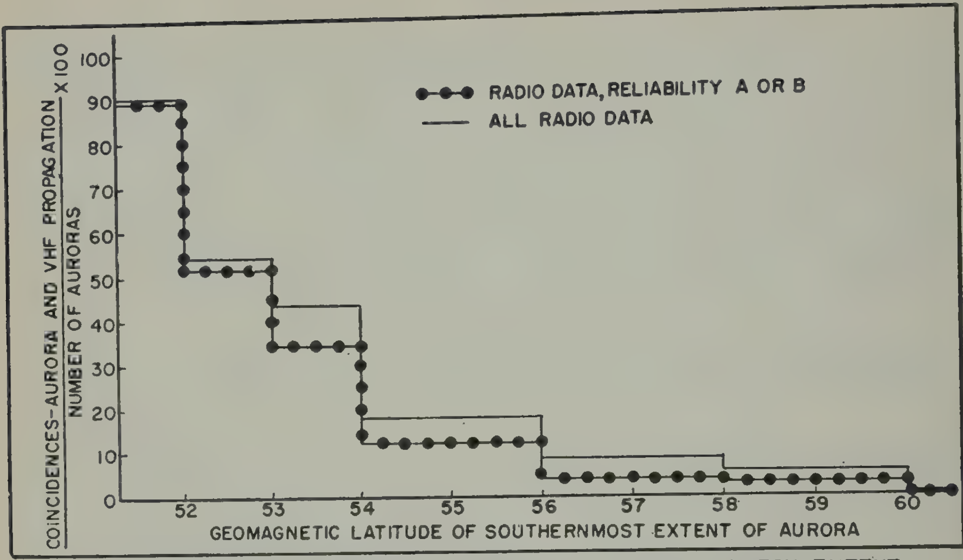

FIG. 1—PERCENTAGE OF DAYS ON WHICH AURORA OF SOUTHERN EXTENT SHOWN WAS ACCOMPANIED BY REPORTS OF "AURORAL PROPAGATION" DEC., 1939—NOV., 1941; DEC., 1945—JUNE, 1950

TABLE 2—*Number of three-hour periods on which "auroral propagation" was reported compared with maximum southern extent of the aurora (only radio data of reliability A or B during non-sunlit periods included), 1939 to 1941 and 1946 to 1950*

Time of day (EST)	Geomagnetic latitude of maximum southern extent						
	< 52°	52°-53°	53°-54°	54°-56°	56°-58°	58°-60°	>60° or none (sunlit periods not counted)
hours							
16 to 19	..	1	..	1	..	..	..
19 to 22	5	2	4	13	5	2	5
22 to 01	3	3	4	3	3	2	3
01 to 04	1	..	..	1	2	..	..
04 to 07	..	..	1	..	..	..	..
	Number of three-hour periods of aurora of indicated maximum southern extent (excluding those on days of "auroral propagation" reports for which time is not known)						
16 to 19	..	2	..	6	9	9	Not determined
19 to 22	5	3	11	63	111	91	1760
22 to 01	3	10	24	80	165	144	1882
01 to 04	4	8	10	47	84	108	2066
04 to 07	2	3	5	33	45	54	Not determined

It can be seen that "auroral propagation" reports were received for many auroras whose maximum southern overhead extent was south of 56° geomagnetic latitude. The extremely small percentage of non-auroral days during which the propagation phenomenon was reported indicates that it is truly connected with the aurora.

A more meaningful study was one in which three-hour periods of auroral propagation were compared with the maximum southern extent during the same period. The results are shown in Table 2 and Figure 2. The operating habits of the radio amateurs show up strikingly in this table. By far the highest percentage of observations was during the period between $19^h\ 00^m$-$22^h\ 00^m$ EST, the evening hours when most of the amateurs are active. A good many amateurs remain active until midnight, and this is reflected in the figures for $22^h\ 00^m$-$01^h\ 00^m$ EST. Outside these two intervals, the reports are quite scattered and meaningless. The reduced number of reports of visual aurora for this period may also be an indication of less observer activity during these hours.

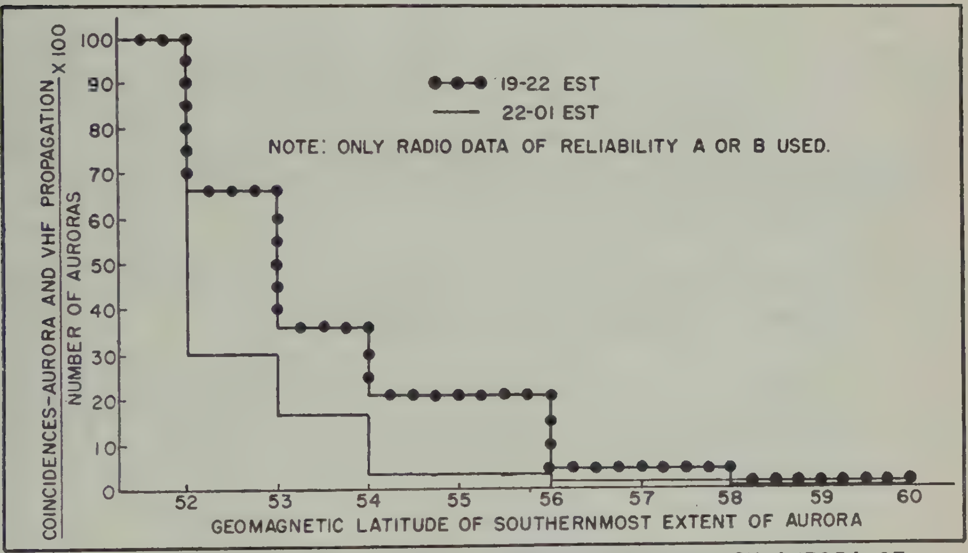

FIG.2–PERCENTAGE OF INDICATED 3-HOUR PERIODS ON WHICH AURORA OF SOUTHERN EXTENT SHOWN WAS ACCOMPANIED BY REPORTS OF "AURORAL PROPAGATION" DEC., 1939–NOV., 1941; DEC., 1945–JUNE, 1950

If we consider only the three-hour period of greatest amateur activity ($19^h\ 00^m$-$22^h\ 00^m$), we see that all auroras overhead south of 52° geomagnetic latitude were accompanied by reports of auroral propagation. More than half of those in the 52°-53° range and smaller but appreciable percentages for the ranges up to 56°, were accompanied by auroral propagation reports. The number of reports for auroras of lesser southern extent is negligible.

Instances of aurora during days when amateur reports failed to list the time of occurrence, and instances of amateur propagation during periods of sunshine or twilight, were omitted from the data in making up this table and Figure 2.

Data for auroral propagation and visual aurora were compared using only the

years 1947 to 1950 to determine whether the results would be appreciably changed because of the better equipment used by the radio amateurs in the post-war era (see

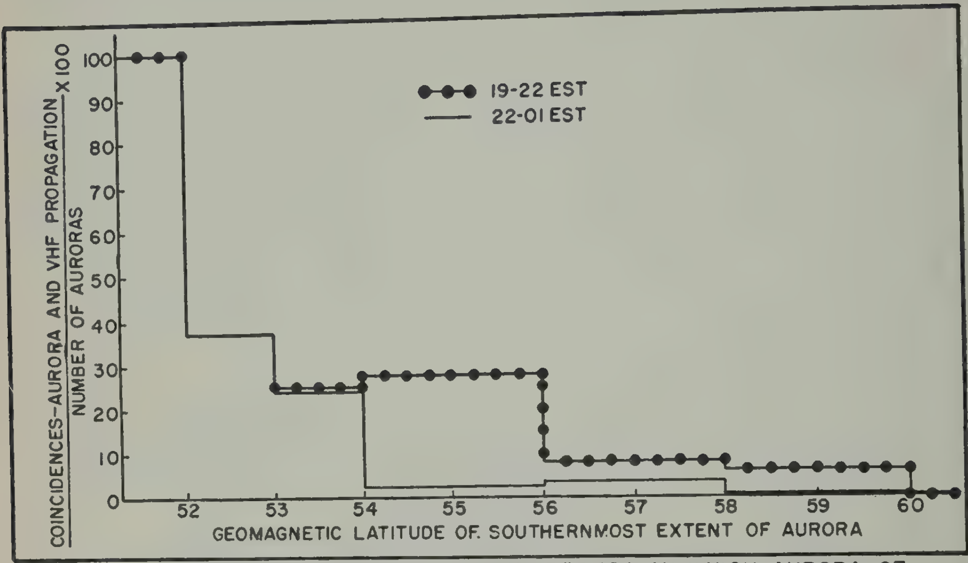

FIG. 3—PERCENTAGE OF INDICATED 3-HOUR PERIODS ON WHICH AURORA OF SOUTHERN EXTENT SHOWN WAS ACCOMPANIED BY REPORTS OF "AURORAL PROPAGATION" JULY, 1947—JUNE, 1950

Table 3 and Figure 3). Results seem to be about the same as when the entire period of observation of the phenomenon by radio amateurs is used.

Table 3—*Number of three-hour periods on which "auroral propagation" was reported compared with maximum southern extent of the aurora (only radio data of reliability A or B during non-sunlit periods included), July 1947 to June 1950*

Time of day (EST)	Geomagnetic latitude of maximum southern extent						
	<52°	52°-53°	53°-54°	54°-56°	56°-58°	58°-60°	>60° or none (sunlit periods not counted)
hours							
19 to 22	2	0	1	9	4	2	0
22 to 01	1	2	3	1	3	1	2
	Number of three-hour periods of aurora of indicated maximum southern extent (excluding those on days of "auroral propagation" reports for which time is not known)						
19 to 22	2	0	4	29	48	33	694
22 to 01	1	8	13	38	96	68	756

Because of the use of beacon stations and the wide-spread reporting network set up by Radio Amateur Scientific Observations [5], more complete data may be available in the future. Because of the small number of large auroras in any given year, this program will have to be continued for several years if it is to give significant results. Since the locations of the observers in this program are known, it may be possible to make a comparison of the southernmost point at which the radio phenomenon is observed with the southernmost point of overhead aurora.

Until more exact data on the fading rate encountered in "auroral propagation" are available, there is little point in attempting to ascribe this phenomenon to some particular cause. Data at hand now indicate, however, that the signals are scattered by some means in the ionosphere, rather than being reflected. If the fading is due to a doppler phenomenon, and if one is to believe the reports of the fading rate, the velocities involved must be considerably higher than those which have been reported for ionospheric winds and turbulence [6].

Conclusions

The phenomenon known to radio amateurs as "auroral propagation" and observed in the frequency range 28 to 148 Mc appears to be truly related to the aurora. This phenomenon is most often observed during periods when overhead aurora is seen at geomagnetic latitudes less than 56°, and it is always observed during auroral displays occurring overhead below 52° during the hours of amateur activity.

This effect is characterized by an extremely high rate of fading which renders voice modulation unintelligible, the need for pointing the beam antennas used toward the auroral display, a lack of any skip effect, and an apparent lack of any change in the polarization of the signal. It is not restricted to hours of visible aurora, for the radio phenomenon is frequently reported during daylight hours immediately preceding dusk and immediately following dawn.

The apparent cause is scattering from something moving at a speed greater than that known for ionospheric winds. A theory to describe adequately the mechanism of "auroral propagation" must await more precise data.

Acknowledgments

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RECENT WORK ON THE RADIOACTIVITY OF POTASSIUM AND SOME RELATED GEOPHYSICAL PROBLEMS*

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ABSTRACT

The half-life now of K^{40} appears fixed at 1.3×10^9 years, with an uncertainty of about 10 per cent. About 12 per cent of the disintegration forms A^{40} by K-capture, while 88 per cent results in Ca^{40} by β-emission. The mean energy released per disintegration is 0.71 Mev, yielding 27×10^{-6} cal/gm.year of ordinary potassium at the present time. With these constants, the production of the A^{40} of the atmosphere, heat production by potassium in the earth, and measurement of age are discussed. Ample quantities of A^{40} have been produced, but release to the atmosphere involves uncertainties. A large part of the heat conducted to the surface of the earth may be generated by potassium decay. The existence of reasonably reliable decay constants should encourage additional efforts to obtain ages of potassium minerals.

Introduction

Since 1930, about one hundred papers or letters have been published on the subject of the radioactivity of potassium, half of them in the last three years. This subject has long been of geophysical interest, but the reported decay constants have undergone remarkable fluctuations. As there is now a convergence of experimental results which appears to settle the points at issue within reasonable limits, it seems worth while to review very briefly the work of the last twenty years on the decay constants, and to reexamine several of the geophysical problems in terms of the new data. These problems are the origin of the argon of the atmosphere, the contribution by potassium to the earth's heat, and the measurement of geologic or cosmic time. At least the first two of these are hardly separable from the broader problem of the history and composition of the interior of the earth, but it is not intended to go beyond a brief survey of some of the implications of the decay constants now believed to be most nearly correct. These constants differ appreciably from the values adopted in several recent reviews of this subject (Jeffreys, 1950; Festa and Santangelo, 1950), which were written before the appearance of the most definitive work.

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Disintegration scheme and decay constants of K^{40}

Though the spontaneous emission of electrons, or β-rays, by potassium was discovered in 1905, it was not until 1937 that the radioactive isotope was definitely identified as K^{40} by Smythe and Hemmendinger, following theoretical discussion of this possibility by Klemperer (1935) and by Newman and Walke (1935), and measurement of the isotopic abundance of K^{40} by Nier (1935), Brewer (1935), and Sampson and Bleakney (1936). Fermi's theory of β-decay (1934) had suggested the likelihood of an additional process, either the emission of a positron from the nucleus, or the capture by the nucleus of an electron from the innermost (K) orbit. (The latter process is known as K-capture, the K referring to the orbit, not to kalium). Møller (1937) showed that K-capture was much more probable than positron emission for heavy elements, contrary to an opinion of Sitte (1935), and later work has failed to find evidence of positrons from K^{40} (Bothe and Flammersfeld, 1941; Bell and Cassidy, 1950). Von Weizsäcker (1937), discussing the possibility of a dual decay of K^{40}, by β-emission to form Ca^{40}, and by K-capture to form A^{40}, proposed the latter process as the origin of the anomalous abundance of A^{40} in the earth's atmosphere (Rankama and Sahama, 1949, p. 776). Supposing that all of the A^{40} of the atmosphere might have been generated by the decay of K^{40} in the earth's crust, von Weizsäcker estimated that about one-third of the disintegrations took place by capture and formation of A^{40}, and predicted that measurable quantities of argon so formed should be found in old potassium minerals.

Bramley (1937) compared the terrestrial abundances of argon and calcium, and estimated that the ratio of captures to β-decays should lie between 1/100 and 1/700, instead of the relatively high value proposed by von Weizsäcker. He also suggested that γ-rays emitted by K^{40} are associated with the capture process rather than with the β-rays; this idea, which now appears to be correct, was based on the momentary equivalence of two figures which have since fluctuated through wide ranges, the rate of γ-ray emission and the supposed branching ratio for argon. Brewer (1937) attacked the problem of the age of the earth, seeking first an upper limit with the assumption that all of the calcium in the crust has been formed by the decay of K^{40}. He also made use of an estimate of heat production by potassium decay to find the age of the crust, but as the disintegration constant used for these computations was about ten times larger than the value which now seems most nearly correct, they are chiefly of historical interest. It is noteworthy, however, that the year 1937 saw the origin of most of the ideas which have dominated this subject ever since.

The first experimental evidence for the suspected process of K-capture was reported in a brief letter to *Nature* in 1943 by Thompson and Rowlands. When an electron of the K-orbit is captured by the nucleus, the orbital vacancy is filled by an electron from an outer orbit with emission of an X-ray quantum characteristic of a line of the K-series of the new nucleus. For argon, the energy of this X-ray is of the order of 3,000 to 4,000 electron volts. Quantitative observation of this feeble radiation is exceedingly difficult, a fact which has given rise to most of the uncertainty about the decay scheme of K^{40}. Thompson and Rowlands estimated the ratio of X-rays to β-rays as about 3 or 4; this would have the effect

of reducing the half-life of K^{40} to one-fourth or one-fifth of the value found for β-decay alone. Coming during the war, this paper had little immediate effect, but another study of the X-rays by Bleuler and Gabriel (1947) resulted in an avalanche of letters to the Editor of the *Physical Review*.

Bleuler and Gabriel gave not only a new value for the ratio of K-captures to β-decays of 1.9, based on measurements of the X-ray intensity, but also a new and high value for the rate of β-decay; the net result was a reduction of the total half-life of K^{40} to 240 million years, almost as low as the sensational figure used by Brewer in 1937. The geophysical importance of such a revision was at once pointed out by Gleditsch and Gráf (1947), and various consequences have been remarked by other writers. It is clear that with so short a half-life, the amount of K^{40} would have declined by a large factor during the earth's history; for example, ten half-lives or 2.4 billion years ago, there would have been 2^{10} or about 1,000 times as much K^{40} as at present. But at present, the radioactivity of K^{40}, rare as it is, accounts for some 10 to 20 per cent of the total radioactive heat generation of the earth's crust, and this in turn is probably a large fraction of the total thermal output. A thousandfold increase of the amount of K^{40} might, therefore, increase the total output by a factor of as much as 100 or 200, and unless the concentration of potassium towards the surface was even more marked at this early date than it is at present, an improbable assumption, the surface thermal gradient would have been increased in a similar ratio. So great an increase of the surface gradient would endanger the stability of the crust, and it would be difficult to understand how rocks of so great an age could have been preserved; on the other hand, a number of age determinations of minerals, by several methods, fall in the range of 2 to 2.4 billion years (Nier, 1939; Nier, et al., 1941; Ahrens, 1947, 1949). Thus a contradiction may be said to exist between this short half-life and the age determinations (Birch, 1947), though the premises cannot be freed from all objections (Jeffreys, 1950).

A similar conclusion that this new half-life must be too short was reached by Poole (1948) on consideration of the relative amounts of argon and potassium in the crust. Verhoogen (1949) has argued that not even the longer half-life suggested by Ahrens and Evans (1948) would permit the existence of a sialic crust as long ago as 3.3×10^9 years, a date found by Holmes (1946, 1947) for the time of consolidation of the crust, and that consequently the sial could not be the source of the lead ores whose isotopic abundances formed the basis of the age calculations.

In the meantime, a number of experimenters had addressed themselves to this problem. The number of titles is now so large that discussion of each one would require undue space; instead, the numerical results since 1930 have been abstracted in Table 1. In many of the papers, new determinations have been combined by the writers with other data in order to compute half-lives and other quantities not completely determined by their own results. Table 1 gives, so far as practicable, only the new contributions in each paper. Until the isotopic abundance of K^{40} was determined, the disintegration constants and half-lives were computed for the whole potassium content; the gamma-activity was commonly given in terms of a radium equivalent. Such quantities, where recomputed, or any values not given by the authors, are enclosed in parentheses.

A number of theoretical papers, not specially noted here, which have contrib-

uted to the understanding of the K^{40} decay, and a few general references, are included in the bibliography.

The observations are of several kinds. There are first the measurements of β-emission, now usually expressed as the number of β's given off per second per gram of ordinary potassium; the same quantity may also be expressed as a disintegration constant λ_β, defined by the usual equation of radioactive decay

$$dN_\beta = \lambda_\beta N\, dt$$

where dN_β is the number of β's emitted in the interval of time, dt, by a number N of K^{40} atoms. Calculation of λ_β requires a knowledge of the isotopic abundance of K^{40} in the potassium used for the experiment. In some of the recent work with

Table 1—*Decay constants for K^{40}*

Year	Authority	β's/sec.gm. K
	(a)—Beta-emission	
1930	Mühlhoff	23
1931	Orbán	(75)
1938	Bramley and Brewer	(26 ± 6)
1947	Bleuler and Gabriel	(53)
1948	Borst and Floyd	23 ± 2
	Gráf	26.8 ± 1.2
	Hirzel and Wäffler	(34 ± 4)
1949	Floyd and Borst	25 ± 2
	Stout	30.6 ± 2.0
1950	Faust	31.2 ± 3.0
	Sawyer and Wiedenbeck (ordinary K)	28.3 ± 1
	Sawyer and Wiedenbeck (enriched to 0.4% K^{40})	30.9 ± 1.7
	Smaller, May, and Freedman	23
	Spiers	30.5
1951	Delaney	32.0 ± 3
Year	**Authority**	E_β(max.),Mev
	(b)—Energy of beta-spectrum	
1933	Bocciarelli	0.22 (mean energy)
1934	Anderson and Neddermeyer	> 0.7
1939	Henderson	1.3
	Libby and Lee	0.72 ± 0.10
1946	Dzelepow, Kopjova, and Vorobjov	1.35 ± 0.05
	Hirzel and Wäffler	1.41 ± 0.02
1947	Henderson	1.3 ± 0.15
1948	Borst and Floyd	1.9 ± 0.2
	Franchetti and Giovanozzi	1.7 ± 0.1
1949	Alburger	1.40 ± 0.03
	Floyd and Borst	1.45 ± 0.15
1950	Alburger	1.36 ± 0.03
	Bell, Weaver, and Cassidy	1.36 ± 0.05
	Feldman and Wu	1.325 ± 0.020
	Smaller, May, and Freedman	1.3 ± 0.1

TABLE 1—Concluded

Year	Authority	γ's/sec.gm. *K*	$\lambda_\gamma/\lambda_\beta$
	(c)—Gamma-emission		
1930	Köhlhorster	(1.5)	....
	Mühlhoff	(1)	....
1934	Gray and Tarrant	....	0.03
1941	Běhounek	(3.8)	....
1947	Gleditsch and Gráf	3.6 ± 0.8	....
1948	Borst and Floyd	3.7	0.16
	Gráf	3.4	0.127 ± 0.012
	Hess and Roll	(2.6)	....
	Hirzel and Wäffler	(3.0 ± 0.4)	0.087 ± 0.012
	Urry	....	0.15
1949	Floyd and Borst	1.2	0.05 ± 0.01
	Sawyer and Wiedenbeck	3.6 ± 0.3	....
1950	Faust	3.6 ± 0.4	(0.115)
	Sawyer and Wiedenbeck	....	0.127
	Smaller, May, and Freedman	....	0.05
	Spiers	3.0	0.10

Year	Authority	E_γ, Mev
	(d)—Energy of gamma-rays	
1934	Gray and Tarrant	2.0
1946	Hirzel and Wäffler	1.54 ± 0.1
1947	Gleditsch and Gráf	1.55 ± 0.05
	Meyer, Schwachheim, and de Souza Santos	1.5
1949	Ceccarelli, Merlin, and Rostagni	1.45 ± 0.15
1950	Bell and Cassidy	1.462 ± 0.01
	Hofstadter and McIntyre	1.48 ± 0.02
	Pringle, Standil, and Roulston	1.47 ± 0.03
1951	Good	1.459 ± 0.007

Year	Authority	Method	λ_K/λ_β
	(e)—Branching ratio for K-capture		
1943	Thompson and Rowlands	X-rays	3 to 4
1947	Bleuler and Gabriel	X-rays	1.9 ± 0.4
	Harteck and Suess	Argon	< 0.1
1948	Ahrens and Evans	Calcium	1.4 ± 0.2
	Aldrich and Nier	Argon	0.02 to 0.09
1950	Ceccarelli, Quarcini, and Rostagni	X-rays	≤ 0.07
	Gráf	γ- and X-rays	0.127 to 0.67
	Inghram, Brown, Patterson, and Hess	A^{40}/Ca^{40}	0.126 ± 0.003
	Sawyer and Wiedenbeck	Auger electrons from X-rays	0.135

sources enriched in K^{40}, the abundance appears to be somewhat uncertain. The gamma-activity may be similarly defined either in terms of the disintegration constant λ_γ, or of the number of gamma's per second per gram of potassium. Finally, the disintegration constant λ_K gives the rate of K-capture or argon pro-

duction. The total disintegration constant λ, which gives the rate of disappearance of K^{40}, is simply the sum of the partial disintegration constants, in this case, the sum of λ_β and λ_K. The half-life of potassium is then $0.693/\lambda$. The practice of computing partial "half-lives" by dividing 0.693 by partial disintegration constants ought to be discouraged.

The energies of the β- and γ-rays are of interest, not only in determining the rate of generation of heat, which is mainly of concern to geophysicists, but also in establishing the decay scheme. One of the questions has been the association of the γ-rays, whether with the β-spectrum or with the K-capture. The β's are now known to have a continuous distribution of energies, from low values to a maximum of about 1.4 Mev (1 Mev = 10^6 electron volts = 3.83×10^{-14} calories), which presumably is the energy difference between the ground states of the K^{40} and Ca^{40} nuclei. The energy of the gamma-rays is close to 1.5 Mev, and, as this exceeds the maximum energy of the β-rays, the gamma-radiation cannot reasonably be ascribed to this reaction. The difference is comparable with the experimental uncertainty of many of the studies, however, and at one point, when high values of the maximum β-energy were reported, the gamma-rays were temporarily assigned to the K^{40}-Ca^{40} reaction. The weight of evidence is now decidedly in favor of the association with the K-capture; aside from the energy difference, efforts to find coincidences between β's and γ's have given negative results (Meyer, Schwacheim, and de Souza Santos, 1947; Smaller, May, and Freedman, 1950). Sailor (1949) has shown by considering a number of other nuclear reactions involving K^{40}, A^{40}, and Ca^{40} that the energy difference $K^{40} - A^{40}$ is slightly greater than the difference $K^{40} - Ca^{40}$, by an amount in good agreement with the difference between E_γ and E_β (maximum). A^{40} is slightly more stable than Ca^{40}.

The amount of gamma-radiation from potassium was seriously underestimated before the work of Běhounek (1941), which was generally thought to be in error because it differed appreciably from earlier work (Table 1). As a consequence, a discrepancy was found by Hess (1946, 1947) between the gamma-radiation observed to arise from the Quincy granite, and that to be expected from the known radioactive content, which had been carefully studied by Evans and Goodman (1941). This was at first interpreted as a new kind of radiation (Hess 1947; Hess and Roll, 1948, *A*), but Urry (1948) demonstrated that on adding to the granite amounts of uranium, thorium, and potassium equal to the original amounts, the radiation was also doubled; the discrepancy could be removed by supposing that about five times as many gamma-rays were produced as had been found by Gray and Tarrant. A new determination of the gamma-radiation by Gleditsch and Gráf (1947) accounted for all of the "surplus radiation" found by Hess and Urry. This value, 3.6 quanta per second per gram of potassium, has since been substantiated by Borst and Floyd (1948), Sawyer and Wiedenbeck (1949), and Faust (1950), and it is close to the 1941 value of Běhounek (3.8). It is worth noting that of the gamma-radiation from granite, in which the ratio of potassium to uranium and thorium usually is lower than in other kinds of igneous rocks, more than one-half of the total is produced by potassium (Gleditsch and Gráf, 1947).

One principal outstanding uncertainty has been the ratio λ_K/λ_β, the ratio of K-captures to β-rays, or of A^{40}/Ca^{40} in the decay products. There are two main lines of attack on this question, as follows: (1) By observation of the X-rays pro-

duced in the capture process, or (2) by analysis for A^{40}. The results from these independent methods were at first irreconcilable. The estimates of Thompson and Rowlands and of Bleuler and Gabriel have already been mentioned. A search by Harteck and Suess (1947) for argon in potassium salts and granite led to λ_K/λ_β less than 0.1, or one-twentieth of the figure given by Bleuler and Gabriel. Aldrich and Nier (1948) found λ_K/λ_β of the order 0.02 to 0.09 by mass-spectrometric analysis of argon extracted from several minerals. Farrar and Cady (1949) were unable to find an increase of the argon/potassium ratio with age of sample, but did not make an isotopic analysis of the argon. On the other hand, Ahrens and Evans (1948) examined the calcium content of several very ancient samples of lepidolite, and by an indirect argument arrived at a value for λ_K/λ_β of 1.4; but again, without isotopic analysis of the calcium.

What now appears to be the most probable solution was given by Sawyer and Wiedenbeck (1950), along with a complete redetermination of all of the decay constants. New values were obtained for the rates of β- and gamma-emission, and in addition, the ratio λ_K/λ_β was found by observing secondary (Auger) electrons associated with the X-rays. As shown by Table 1, the two ratios $\lambda_\gamma/\lambda_\beta$ and λ_K/λ_β are identical, within the margins of error, and equal to about 0.13. It was concluded that both are measures of the rate of K-capture; the number of gammas equals the number of captures or the number of A^{40} atoms produced. This conclusion, though with different values, had already been reached by Hirzel and Wäffler (1948) and by Suess (1948).

Further confirmation for this value has since appeared. Ceccarelli, et al. (1950) obtained about 0.07 from an improved method of X-ray counting in a magnetic field which eliminated the strong unwanted background of β-rays. Inghram and coworkers (1950) found the ratio of radiogenic A^{40}/Ca^{40} by an ingenious method of isotopic dilution and analysis with a mass spectrometer; their value, 0.1263 ± 0.0027, is in good agreement with those of Sawyer and Wiedenbeck. It appears that the branching ratio for argon has been definitely limited to the range 0.1 to 0.15, with the most probable value close to 0.13, and that this is also the ratio of γ- to β-rays. This means that of a given number of disintegrating K^{40} atoms, 12 per cent will decay to A^{40} and 88 per cent to Ca^{40}, as shown by the diagram of Sawyer and Wiedenbeck, reproduced as Figure 1.

With the branching ratio established, the half-life depends upon the absolute rate of β-decay. Of the 14 determinations in Table 1, two (75 and 53) are certainly erratic; of the other 12, seven fall in the range 28 ± 3, and the mean of 12 is 27.8. On the other hand, the best single value is probably 30.6 ± 2.0 (Stout 1949), which is closely supported by the determinations of Faust, Spiers, and Delaney, and by Sawyer and Wiedenbeck for an enriched sample (30.9 ± 1.7). There might be some reason, therefore, for a slight revision of the value preferred by Sawyer and Wiedenbeck (28.3 ± 1), but as these writers have given a complete and self-consistent solution which does not differ significantly from the other most consistent values, it does not seem worth while to tamper with their results.

The calculation of the disintegration constants proceeds as follows: The number of β's per second per gram of (ordinary) potassium is 28.3. The atomic abundance of K^{40} is 1.19×10^{-4} (Nier, 1950). The atomic weight of K^{40} is, of course, close to 40, while the atomic weight of ordinary potassium (mostly K^{39}) is 39.096; the

abundance ratio of K^{40}, by weight, is thus 1.22×10^{-4}. We have 23.3×10^4 β's per second per gram of K^{40}, which contains $6.064 \times 10^{23}/40$ atoms. Then $\lambda_\beta = 23.3 \times 10^4 \times 40/6.064 \times 10^{23} = 1.54 \times 10^{-17}$ per second, or 0.483×10^{-9} per year. With $\lambda_K/\lambda_\beta = 0.135$, $\lambda_K = 0.065 \times 10^{-9}$ per year, and the total disintegration constant, $\lambda = \lambda_K + \lambda_\beta = 0.548 \times 10^{-9}$ per year. The half-life of K^{40} is then $(1.27 \pm 0.06) \times 10^9$ years, according to Sawyer and Wiedenbeck. The real uncertainty seems more likely to reach about 10 per cent, in view of the scatter of the values of λ_β mentioned above.

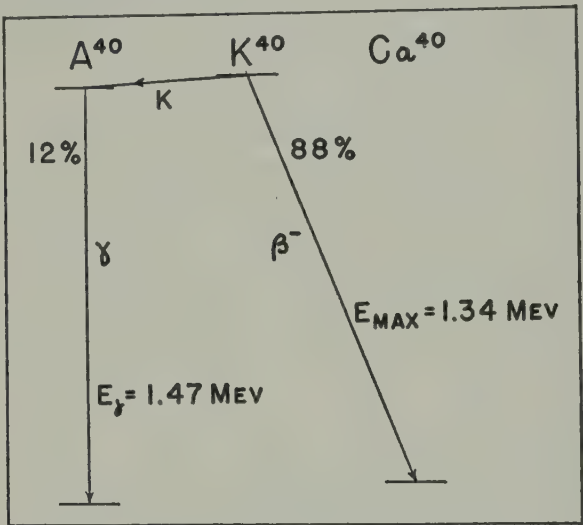

FIG. 1—DECAY SCHEME FOR K^{40}, AFTER SAWYER AND WIEDENBECK; ENERGIES AFTER ALBURGER

We are now able to calculate the rate of energy release. In one second, a gram of ordinary potassium produces 28.3 β's and 3.6 gammas. The energy of the gamma-rays is 1.5 Mev, with an uncertainty of not more than 5 per cent. The β-rays have a continuous distribution of energy to a maximum of 1.34 Mev (Henderson, 1939; Dzelepow, et al., 1946; Alburger, 1950; Bell, Weaver, and Cassidy, 1950; Feldman and Wu, 1950). The mean energy of normal β-ray distributions is roughly one-third of the maximum, but K^{40} exhibits one of the theoretically "forbidden" types, with somewhat greater concentration toward high energies than is usual. Alburger (1951) has recalculated the mean energy of this distribution, finding 0.605 ± 0.010 Mev, somewhat higher than the result of Marinelli, et al. (1947), based on less complete data. The total per second per gram of potassium is $28.3 \cdot 0.60 + 3.6 \cdot 1.5 = 22.4$ Mev $= 8.6 \times 10^{-13}$ cal/sec.gm.K $= 27 \times 10^{-6}$ cal/year.gm.K. This is equivalent to 0.22 cal/year.gm.K^{40}. For comparison, the energies associated with the uranium and thorium series are 0.72 cal/year.gm.U and 0.20 cal/year.gm.Th. The uncertainty for K^{40} should not exceed about 10 per cent. The mean energy per disintegration of K^{40} is, similarly, $0.88 \cdot 0.60 + 0.12 \cdot 1.5 = 0.708$ Mev $= 0.27 \times 10^{-13}$ cal.

Earlier estimates of energy release from potassium are recorded in Table 2. Holmes and Lawson (1926, 1927) first called attention to the importance of radio-

active heating by potassium, but though the mean energy of the β-rays, as estimated by Hahn, was nearly correct, the half-life available at that time was much too short. Their figure (120×10^{-6} cal/year.gm.K) was used by Jeffreys in the 1929 edition of "The Earth." By 1933, however, as a result of Mühlhoff's relatively good determination of the β-emission (Joly, 1930), Holmes had corrected this to 20×10^{-6}, close to the latest value. Evans and Goodman (1941), Urry (1941), and Bullard (1942) were influenced by the low values of energy reported by Bocciarelli (1933) and apparently overlooked the abstract by Henderson (1939). The work of Bleuler and Gabriel led to a precipitate increase to 38×10^{-6} (Gleditsch and Gráf, 1947). Several other estimates, together with the current half-lives, are shown in Table 2.

TABLE 2—*Energy production and half-life of potassium*

Date	Authority	Energy release in 10^{-6} cal/year.gm. K	Half-life of K^{40} in 10^9 years
1926	Holmes and Lawson	124	0.18
1933	Holmes	20	1.6
1941	Evans and Goodman, Urry	5 ± 2	1.6
1942	Bullard	8	1.5
1947	Gleditsch and Gráf	38	0.24
1948	Ahrens and Evans	26	0.45
1948	Borst and Floyd	20	0.4 to 1.2
1948	Gráf	22 ± 3	0.5
1950	Jeffreys	22.6 ± 3.6	0.49 to 0.8
1951	Alburger, Sawyer and Wiedenbeck	27 ± 3	1.27 ± 0.1

The argon problem

Argon constitutes about 1.3 per cent of the atmosphere; the total mass of argon in the atmosphere is 0.66×10^{20} gm (Rankama and Sahama, 1949, p. 305), of which 99.6 per cent is A^{40} (Nier, 1950). In the atmosphere, argon is about 1,000 times as abundant as neon, and 4,000 times as abundant as krypton (Rankama and Sahama, pp. 771, 774-776), although neon appears to be cosmically much more abundant. It was von Weizsäcker's suggestion that the A^{40} has been evolved by decay of the K^{40} of the earth's crust. From the amount of K^{40} now present, say N_0, and the disintegration constant λ, we find the amount N at any time t years ago as $N = N_0e^{\lambda t}$. The amount of K^{40} which decays in this time is $N_0(e^{\lambda t} - 1)$, and the amount of argon formed, $0.12\, N_0(e^{\lambda t} - 1)$. (As these atoms are all of mass 40, nearly enough, N and N_0 may be expressed either as numbers of atoms or as masses). For the "age of the earth," let us take Holmes' estimate of 3.3×10^9 years (Holmes, 1946, 1947; Houtermans, 1947; Bullard and Stanley, 1949; see, however, Jeffreys, 1948, 1949, Holmes, 1949). Then with $\lambda = 0.55 \times 10^{-9}$/year, we find $0.12\,(e^{\lambda t} - 1) = 0.61$; the mass of A^{40} generated in 3.3×10^9 years is 0.61 times the mass of K^{40} remaining.

The total mass of argon so produced depends, of course, on the total mass of potassium. A relatively good guess can probably be made for the crust on the basis of the average composition of "igneous rocks," as compiled by Goldschmidt

or by Clarke and Washington (Rankama and Sahama, p. 39). The average value for potassium is 2.6 per cent, about midway between the values for granites, 4 to 5 per cent, and for basalts and gabbros, about 1 per cent. For the mean thickness of crust having this average potassium content, let us take 33 km; because of the difference between the continental and oceanic crusts, this value is debatable, but the order of magnitude should be right.

Below the crust is the "mantle," a layer probably composed mainly of ferromagnesian silicates, extending to the boundary of the "core" at a depth of 2,900 km. This layer comprises about two-thirds of the mass of the earth, as contrasted with less than one per cent represented by the crust. The total potassium content of the mantle will be comparable with that of the crust even if the average proportion of potassium in the mantle is only a few hundredths of one per cent. Though we may feel fairly sure that the major components of the mantle are magnesium, silicon, iron, and oxygen, there can be little confidence about the content of minor elements such as potassium. At present, the two most popular theories concerning the chemical composition of the mantle are (1) that it resembles the ultrabasic rock dunite, composed mainly of magnesian-rich olivine, or (2) that it resembles the average stony meteorite, a more complex mineral

Table 3—*Argon generation in 3.3 $\times$ 10^9 years, and heat production by potassium at present*

Region	Total mass	K	K	K^{40}	A^{40}	Heat
	10^{24} gm	per cent	10^{24} gm	10^{20} gm	10^{20} gm	10^{20} cal/year
Ocean	1.4	0.038	0.0005	0.0006	0.0004	....
Sedimentary rocks	~1	3	0.03	0.03	0.02	....
Crust	47	2.6	1.2	1.5	0.9	0.32
Mantle (dunite)	4000	0.03	1.3	1.6	1.0	0.35
Mantle (meteorite)	4000	0.17	6.8	8.3	5.1	1.8

assemblage with a higher proportion of minor constituents. We may question whether either of these materials should be taken as representing exactly the mean chemical composition of the mantle; nevertheless, they furnish two values for the potassium content sufficiently different to lend interest to the computation. Daly (1933, p. 20) gives the average potassium content for 10 dunites as 0.033 per cent; the average for the silicate phase of stony meteorites is 0.20 per cent (Brown, 1949). With these figures, we find the total quantities of argon produced by the potassium now attributed to various regions, during the time of 3.3 $\times$ 10^9 years (see Table 3). It has been supposed that the stony meteorites represent the composition of the mantle plus crust *before* differentiation, whereas the dunite is taken to be a product of differentiation from a primitive composition which would have had about 0.06 per cent of potassium.

The argon production of the potassium in the ocean and the sedimentary rocks is negligible, even if we allow for large errors in the estimates (Rankama and Sahama, pp. 221-224). The argon of the atmosphere, 0.66 $\times$ 10^{20} gm, amounts to more than two-thirds of all the argon generated in the crust, as here defined, during

the time of 3.3×10^9 years. While the total quantities are more than ample, we have to face the question of how much of the argon, generated in crust or mantle, can be expected to have escaped into the atmosphere.

If we suppose that retention of argon became possible 3.3×10^9 years ago, then it is easy to show that a total amount of argon equal to that now in the atmosphere would have been generated in the first 500 million years, on the dunite hypothesis, or the first 150 million years, on the meteorite hypothesis; or we may say that, in either case, of the total radiogenic argon now present, about 50 per cent was generated in the first 1,000 million years, and 80 per cent in the first 2,000 million years. We may suppose that the crust has been able to release nearly all of its argon, either by solution or fusion, or that a considerable fraction of the argon of the mantle has been able to make its way to the atmosphere. Conceivably vulcanism and intrusion on a vast scale characterized the first 10^9 years and permitted the release of much of the argon then available. Suess (1948) has even proposed that a large part of the earth's surface was "in a molten liquid state during the long period from 33×10^8 until 20×10^8 years ago." The difficulty of supplying enough heat for such a long period of high-temperature radiation seems insuperable; by contrast we have Jeffreys' estimate of 10^4 years for the time required for complete solidification (1929, p. 79; see, however, pp. 147-148).* It is clear that the argon problem, like that of the development of the rest of the atmosphere and hydrosphere, is inseparable from the unsolved question of the origin and evolution of the earth.

As a number of writers have shown (Högbom, 1894; Chamberlin, 1947; Brown, 1947; Rubey, 1950), there is evidence that large quantities of water and carbon dioxide have escaped from the interior throughout the history of the earth. The formation of the crust may have gone on steadily by intrusion, lava flows, "granitization," and other processes, by which water and other components are brought to or near the surface. It has been remarked that the mass of the ocean is about 3 per cent of the mass of the crust, a reasonable relationship on the hypothesis that all of this water was once dissolved in the crust (Ingerson, 1950). Another noteworthy fact, or coincidence, is the relation between areas of Pre-Cambrian granite and of younger granite in the United States. Daly (1933, p. 35) gives figures for two small but relatively well-mapped areas, the Pacific Cordillera and the Appalachian belt, as follows:

	Pacific Cordillera (square miles)	*Appalachian belt* (square miles)
Pre-Cambrian granite	2,089	1,151
Younger granite	402	194
Ratio of areas, Pre-Cambrian/younger	5.2	5.9

Now the ratio of Pre-Cambrian time to all later time is, with Holmes' figure, 2,800/500, or 5.6. These samples, so far as they go, suggest the thesis of a uniform rate of intrusion or formation of granite; it would be of great interest to have

*Prof. Jeffreys has kindly informed me that he now believes this time may be as long as 10^6 years.

similar figures for large areas. Without pressing any interpretation, however, let us consider a simple problem arising from these comparisons.

Let us suppose that the rate of introduction of potassium into the crust is uniform, so that in every interval of time, dt, a mass of potassium, dM = c dt, is added to the crust, with c constant. In the process, let the accumulated argon in this element of mass be released to the atmosphere. We take the mass of potassium in the crust and the mass of argon in the atmosphere as zero at the time $t = 0$, when the proportion of K^{40} in the potassium was k, say. Then at time t, the proportion would be $ke^{-\lambda t}$, and the mass dM will contain a mass of argon, dA = 0.12 k $(1 - e^{-\lambda t})$ dM. The increase of argon in the atmosphere is then dA = 0.12 kc $(1 - e^{-\lambda t})$ dt; this leads to the total mass of argon in the atmosphere as function of time, $A(t) = (0.12\ kc/\lambda)\ (\lambda t - 1 + e^{-\lambda t})$. The total generation of argon in this time, including what remains in the crust, is 0.12 kM $(1 - e^{-\lambda t})$; $M = ct$, so that the ratio of argon released to argon generated is

$$\frac{\lambda t - 1 + e^{-\lambda t}}{\lambda t(1 - e^{-\lambda t})}$$

This quantity does not vary greatly with time; its limiting values are 1/2, for $t = 0$, 1 for t infinite. For $t = 3.3 \times 10^9$ years, its value is 0.64. In order to have released 0.66×10^{20} gm of A^{40} in this time, a total of 1.03×10^{20} gm must have been generated. The agreement between this figure and the 0.9×10^{20} gm given in Table 3, based on the estimate of potassium content of the whole crust, is doubtless closer than it deserves to be, in view of the crudity of the theory; it does appear, however, that the argon content of the atmosphere is roughly consistent with a uniform increase of potassium content of the crust throughout its history, with the potassium introduced in such a form that the argon then associated with it could escape. The enrichment in potassium may have been accompanied by a concentration of the other radioactive elements, and by release of water and carbon dioxide. It would be useful to have a means of testing a similar hypothesis of linear increase of water and carbon dioxide with time. Another possible history for the argon is suggested below.*

Houtermans and Jordan (1946) have sought to test a physical or cosmological theory which predicts a variation of the rate of β-decay with time (variation of disintegration constant) by an appeal to the decay of K^{40} as indicated by the abundance of argon. The argon problem is itself beset with too many uncertainties to lend much value to such a test.

Heat generation by potassium

The relative importance of potassium in the radioactive generation of heat is illustrated by typical figures for several kinds of rock (Table 4). Potassium accounts for about one-sixth of the total heat generation in granites, about one-third in dunite, with Davis' (1947) value for radium in dunite, and the assumption that thorium contributes an amount equal to that of the uranium series. No reliable figures are available for the uranium or thorium content of the stony meteorites.

**Note added in proof:* For further discussion of this problem, see Tatel (1950), and Kulp (1951).

Recent work on ultrabasic rocks and iron meteorites by Davis (1947, 1950) and on iron meteorites by Arrol, Jacobi, and Paneth (1942) has led to radium contents for these materials about one-tenth as high as those current only a few years ago. It seems highly probable that a comparable reduction for the stony meteorites will occur when these particularly interesting materials are studied with the best available techniques. Davis has given a few preliminary values for the silicate portion of meteorites which are even lower than the values for the irons. Urry's conclusion (1949) that the stony meteorites contain more radium than dunite is based on a comparison of revised values for dunite with unrevised values, probably too high, for the meteorites.

Figures for rates of heat production by potassium alone, in the crust and mantle, are included in Table 3; they refer to the present time, with K^{40} taken as 1.2×10^{-4} of all potassium, by weight. In the past, these quantities would have been greater, by a factor reaching 6.1 at a time of 3.3×10^{9} years ago. In the past, potassium was also relatively more important, as U^{238} and Th have much longer half-lives, and U^{235}, with a shorter half-life, is much less abundant.

TABLE 4—*Radioactive heat generation in typical rocks*

Kind of rock	Concentration, gm/gm			Heat production, in 10^{-6} cal/gm.year			
	$U \times 10^6$	$Th \times 10^6$	K	U	Th	K	Total
Granite	4	13	0.04	2.9	2.6	1.1	6.6
Basalt	1	3	0.012	0.7	0.6	0.3	1.6
Dunite	0.014	?	0.0003	0.01	(0.01)	0.01	0.03
Stony meteorites	.....	..	0.002	....	....	0.05	....

If we suppose that the flow of heat in the oceans is of the same order as in the continents (Birch, 1950), then the present rate of conduction of heat to the surface of the earth may be estimated as about 2×10^{20} cal/year. The determination of heat flow in the ocean bottoms is still in a preliminary stage (Pettersson, 1949; Benfield, 1950; Scripps Institute exhibits at the meeting of the Geological Society of America in Washington, 1950), and there are far too few determinations in the continents, so the uncertainty in this estimate may easily reach 50 per cent or more. Estimates of the loss of heat by other processes are even less reliable, ranging from an order of magnitude smaller than the conducted heat (Gutenberg, 1939, p. 159) to "many times" the conducted heat (Bowen, 1928). Losses from hot springs, lava flows, and volcanoes vary with time and place in a capricious way, and a long-term average is not easily obtained. If, in accordance with the suggestion which has been developed above to account for the atmospheric argon, we suppose that the ocean and the crust have originated over a period of 3.3×10^{9} years by intrusion and vulcanism, we obtain average values of thermal loss of 3×10^{17} cal/year for the water, and 7×10^{18} cal/year for the crust. It has been assumed that the water emerged as steam, with an average release of 600 cal/gm, and the crust as lava or magma, with an average release of 500 cal/gm, probably

an overestimate. Much of the water in thermal areas may be recirculated surface water, heated by the steam just considered or by a fraction of the conducted heat. Let us proceed on the assumption that the total loss of heat is of the same order as the conducted heat.

About one-half of the conducted heat lost in the continents is readily accounted for in terms of the uranium and thorium of the crust alone, largely concentrated in the granitic rocks. A similar proportion may hold for the oceans, though with these elements distributed through a greater depth; this is no more than a guess. About 10^{20} cal/year remain for potassium and any other sources, such as initial heat. This is of the same order as the figures for the output from potassium alone (Table 3). Probably no significant discrepancy can be demonstrated for either of the assumptions regarding the mean potassium content of the mantle, even though the figure for potassium alone, on the meteorite hypothesis, is about twice as large as the remainder assigned to potassium plus initial heat; the uncertainties are too great to warrant closer interpretation. It is clear, however, that the radioactivity of K^{40} provides a substantial fraction of the thermal output of the earth, and there is an indication that the mean potassium content ought not to exceed about 0.1 per cent.

In recent discussions of the origin of the solar system (Von Weizsäcker, 1944; Schmidt, 1944, 1945; Chandrasekhar, 1946; Hoyle, 1946; Spitzer, 1946; Whipple, 1946; Ter Haar, 1948; Edgeworth, 1949; Latimer, 1950), it has been postulated that the earth and other planets developed as centers of accumulation in a cosmic dust-cloud. This theme has been developed with many variations, but the details still remain vague; the time during which the major accumulation took place has been variously estimated as from 10^5 to 10^9 years. In the latter case, much of the accumulation would have an initial temperature below the melting point. On examining the subsequent history of a cold accumulation, Urry (1949) concluded that "if the time that has elapsed since the solidification of the earth is three billion [10^9] years, the time prior to this necessary to raise the temperature from 0°C and to melt the interior [by radioactive heating] would be of the order of three to four billion years." This was based on a mean potassium content of 0.013 per cent and the radium content of Davis' dunite. Latimer (1950) has estimated a heat production of 200 cal/gm in the "first" 1.5×10^9 years from K^{40} alone. It is interesting to repeat these calculations with the new values for potassium.

The amounts of K^{40} existing at earlier epochs, relative to the present amount, the heat produced per gram of rock during certain intervals, and also the masses of argon produced during these intervals, are given in Table 5 for two values of mean potassium content of the mantle, 0.06 per cent and 0.20 per cent, as before. The potassium is supposed to be uniformly distributed throughout the mantle; there is as yet no crust. The total heat of melting from 0°C may be taken as between 400 and 600 cal/gm; the latent heat of melting, at the melting point, is about 100 cal/gm. In the deeper parts of the mantle there would be little loss of heat by conduction, even in the long periods considered in Table 5.

These figures show that potassium alone would have sufficed to cause melting in about 10^9 years, prior to 3.3×10^9 years ago, for the higher concentration; or about 2×10^9 years for the lower one. With allowance for uranium and thorium, or for a higher initial temperature (Ter Haar and Wergeland, 1947; Benfield,

1950), these times would be shortened; it is also likely that a stage of mobility would be reached before melting reached completion. Thus it seems highly probable that even an initially "cold" accumulation of undifferentiated material having a potassium content of 0.06 per cent or more must pass through a stage of at least partial liquefaction, and that if we date the "resolidification" at 3.3×10^9 years ago, the time prior to this required for melting would be less than 10^9 years, and perhaps not much longer than the rather indefinite time of accumulation.

Another possible solution to the argon problem is indicated by the figures of Table 5; namely, that a moderate degree of outgassing took place during a "prehistoric" period of some 500 million years.

TABLE 5—*Production of heat and argon by decay of K^{40} during certain "prehistoric" periods*

Time before present, in 10^9 years	Mass of K^{40} relative to present value	Decay of K^{40}, relative to present mass	Heat production, cal/gm		A^{40} production, in 10^{20} gm	
			0.06 per cent K	0.20 per cent K	0.06 per cent K	0.20 per cent K
0	1					
		5.1	150	500	1.8	5.9
3.3	6.1					
		2.9	85	285	1.0	3.3
4.0	9.0					
		2.9	85	285	1.0	3.3
4.5	11.9					
		3.7	110	365	1.3	4.3
5.0	15.6					

Measurements of age

There have been several efforts to use the radioactive decay of potassium to determine the time of various remote cosmic events. Brewer (1937) sought an upper limit for the age of the earth's crust on the assumption that all of the calcium of the crust has been produced by decay of K^{40}. Figures for the average composition of igneous rocks (Rankama and Sahama, p. 39) show nearly equal amounts of calcium and potassium; since Ca^{40} comprises about 97 per cent of all terrestrial calcium, the Ca^{40}/K^{40} ratio is of the order of 10^4. With Brewer's 1937 value of the half-life of K^{40}, about 170 million years, all of the calcium would be generated in little more than 2×10^9 years. In 1938, however, with the good value of half-life found by Bramley and Brewer, this figure was raised to 15×10^9 years (Brewer, 1938), and is not greatly different with the present half-life. A similar calculation for the average stony meteorite, for which the Ca^{40}/K^{40} ratio is about seven times larger than for the average igneous rock, leads to 21×10^9 years. As a large and indeterminate fraction of the calcium is doubtless primitive, the value of these results is questionable, so long as no contradictions arise. It would evidently be embarrassing to find an age by this method of a few hundred million years.

We can turn this question around and inquire how much calcium has been produced by the decay of K^{40} during 3.3×10^9 years. The figures for radiogenic

calcium corresponding to the various potassium contents adopted in Table 3 are 0.88/0.12, or 7.3 times the respective amounts of A^{40}. The fraction of the calcium in either crust or mantle that can have been produced by decay of its own potassium in this time is so small as to change the average isotopic abundance of Ca^{40} by no more than 4×10^{-4} (for the crust).

Brewer's method has recently been applied to the meteorites by Festa and Santangelo (1950), but with a short half-life of 350 million years, which leads to an upper limit of 6.4×10^9 years. These writers also attempt to circumvent the troublesome question of primitive calcium by a comparison of the relative abundances of calcium and potassium in the average igneous rock and the average stony meteorite. They obtain the promising result of 3×10^9 years for the age of the earth, but, aside from the too-short half-life, their method should have led to an age of zero had there been no numerical inconsistency. They suppose that the rocks and meteorites initially had the same, though unknown, isotopic abundance of Ca^{40}, and they are forced to suppose, lacking data to the contrary, that the Ca^{40} content is now 97 per cent of the calcium in both; these assumptions are inconsistent with different potassium contents except at the time zero.

Suess (1938, 1939) and Schumb, Evans, and Leaders (1941) compared the beta-activity of samples of meteoritic and terrestrial potassium, and could find no significant differences. This method is limited in sensitivity, however, and differences of as much as 5 or 10 per cent might have been missed. Smythe (1939) made a similar comparison of potassium from granite from a depth of 2,500 feet with surface samples; at this time, there was a question of formation of K^{40} from K^{39} by neutron capture near the surface of the earth, but no difference of activity could be detected. Brewer (1939), using a mass-spectrometer, could find no difference in abundance of the potassium isotopes in fresh Vesuvius lava or clay soils; a beta-counting experiment with Saratoga Springs water also gave a negative result. As matters now stand, we have no reason to doubt that all potassium of which we have samples originated at the same time with a universal initial isotopic distribution, but the negative evidence on which this conclusion depends is not of the highest precision.

Though not bearing directly upon the abundance of K^{40}, two studies of the isotopic ratio K^{39}/K^{41} are of interest. K^{39} is approximately 14 times as abundant as K^{41} (see White and Cameron, 1948; for a review of these determinations, also Nier, 1950). Brewer (1936) examined this ratio for a number of minerals and plants, finding values for plants ranging from 12.6 to 14.3. Cook (1943) could not confirm this variation, and it seems likely that the experimental errors have generally been somewhat larger than those estimated by the investigators. It would be difficult to establish with certainty a variation of abundance of K^{40} of less than perhaps 5 per cent.

The existence of reliable decay constants for potassium will undoubtedly stimulate efforts to determine the age of formation of individual potassium minerals, either by the A^{40}/K^{40} ratio or the Ca^{40}/K^{40} ratio. Contamination by primitive calcium or by atmospheric argon can be dealt with by use of the mass-spectrometer, at least so long as the amounts are not excessive. It remains to be seen how serious will be such disturbances as leakage of argon or chemical alteration of the potassium/calcium ratio. The results of Aldrich and Nier suggest that leakage of argon

may be appreciable. Table 6 shows their values of the A^{40}/K^{40} ratio, the supposed ages, and the ages found from the relation, $A^{40}/K^{40} = 0.12 (e^{\lambda t} - 1)$, with $\lambda = 0.55 \times 10^{-9}$/year.

TABLE 6—*Age of minerals by the A^{40}/K^{40} ratio, after Aldrich and Nier*

Mineral	A^{40}/K^{40}, gm/gm	Age, in 10^9 years	
		Supposed	Calculated
Orthoclase	0.07	14	8.3
Microcline	0.008	3.5	1.2
Sylvite	0.0013	2.0	0.2
Langbeinite	0.009	2.0	1.3

Except for the sylvite, the orders of magnitude are in agreement. A reexamination of the geologic ages assigned to these minerals might be warranted. Sylvite has, of course, by far the highest proportion of potassium; it is also easily recrystallized. On the other hand, the sylvite studied by Inghram and others apparently retained substantially all of its argon.

Suess (1950) could find less than 10^{-6} cm^3 of argon per gram in tektites containing 1 to 2 per cent of potassium, though the total gas content was as much as 0.1 cm^3/gm, and concluded that they must be much younger than the solar system.

Summary

After some remarkable episodes, the half-life of K^{40} seems now to be reliably fixed at 1.3×10^9 years, with an uncertainty no greater than about 10 per cent; this is close to the 1938 value of Bramley and Brewer. A^{40} is formed by K-capture in about 12 per cent of the disintegrations, Ca^{40} by β-emission in 88 per cent. One γ-ray of 1.5 Mev, and one soft X-ray, accompany each capture process. The maximum energy of the β-spectrum is 1.34 Mev, the mean energy 0.60 Mev. The mean energy release per disintegration is 0.71 Mev, corresponding to 27×10^{-6} cal/gm.year of ordinary potassium at the present time.

These constants have been employed for a brief review of the following problems: The production of the A^{40} of the atmosphere, heat production by potassium in the earth, and the measurement of age. Ample quantities of A^{40} have been produced, but the question of release to the atmosphere encounters the uncertainties attending the history of the earth's crust and interior. A simple theory of steady enrichment of the crust in potassium gives a good account of the ratio of argon in the atmosphere to estimated potassium content of the crust, but a number of other interpretations are conceivable. A large, possibly the greater, part of the heat conducted to the surface of the earth, may be generated by the decay of potassium. A uniform earth with a mean potassium content of the order of 0.1 per cent would eventually pass through a stage of at least partial liquefaction, even if initially cold; the time required might not greatly exceed 10^8 years. Few

determinations of age have as yet been made by use of the radioactivity of K^{40}; but the existence of reasonably reliable constants should encourage efforts to obtain ages of potassium minerals.

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GEOMAGNETIC AND SOLAR DATA

INTERNATIONAL DATA ON MAGNETIC DISTURBANCES, THIRD QUARTER, 1950

Preliminary report on sudden commencements

S.c.'s given by five or more stations are in italics. Times given are mean values, with special weight on data from quick-run records.

1950 July 01d 21h 54m: Wn El.—03d 08h 42m: Ka.—04d 00h 33m: El.—05d 22h 17m: El.—07d 01h 53m: Wn.—09d 12h 34m: El.—09d 18h 14m: Pi.—11d 00h 17m: El.—11d 13h 35m: SJ Hu.—11d 13h 58m: Ka El.—11d 17h 58m: Wn.—11d 18h 08m: Do Es Wn.—11d 18h 35m: Es Wn Ma El.—11d 21h 28m: Wn El.—12d 03h 10m: Do.—13d 09h 05m: El.—13d 16h 26m: El.—13d 19h 41m: El.—15d 01h 18m: Am.—16d 21h 43m: So.—19d 16h 46m: El.—20d 05h 38m: Te El To.—21d 19h 53m: So.—21d 20h 02m: Wn.—21d 20h 39m: So Wn.—*24d 01h 51m:* thirty-one.—24d 09h 40m: El.—24d 11h 05m: El.—24d 16h 57m: El.—27d 09h 51m: Wi El.—28d 02h 41m: El Am.—30d 20h 21m: Wn.—31d 22h 00m: Do Wn El.

1950 August *02d 19h 10m:* Do So Wn Ma El.—05d 18h 50m: Wn.—05d 23h 26m: Wn Tl El.—06d 10h 51m: Do ZS.—06d 20h 12m: Tl El.—06d 21h 46m: So.—07d 03h 41m: Sw El To Am.—*07d 10h 55m:* twenty-five.—08d 22h 24m: Wn.—08d 23h 15m: El.—09d 00h 06m: Am.—09d 20h 19m: Do.—09d 23h 13m: Wn.—10d 19h 00m: El.—11d 21h 08m: Tl.—*14d 08h 21m:* seven.—14d 12h 27m: Sw.—14d 12h 40m: Tl.—14d 21h 45m: Es Ab El.—16d 01h 53m: Am.—17d 01h 42m: Am.—17d 02h 19m: Am.—18d 11h 38m: Wn.—*18d 15h 38m:* twenty-one.—18d 17h 57m: Si.—18d 20h 41m: Wn El.—*19d 10h 06m:* seventeen.—19d 19h 30m: Ap.—20d 10h 18m: Hr.—21d 01h 55m: El.—21d 08h 20m: El.—23d 23h 05m: Wn.—24d 22h 02m: So Wn.—27d 23h 00m: Te.—28d 01h 37m: Wn.—*28d 10h 20m:* seventeen.—29d 23h 36m: El.—30d 01h 10m: Sw.—31d 08h 09m: El.

1950 September 02d 18h 50m: Wn.—02d 19h 34m: Do.—03d 02h 06m: Do.—03d 08h 15m: El.—03d 10h 50m: El.—03d 19h 03m: El.—03d 23h 34m: Tl El.—04d 18h 04m: So.—04d 22h 23m: Wn.—05d 06h 15m: Tl.—05d 16h 25m: El.—05d 18h 42m: So Do El.—05d 19h 42m: El.—05d 21h 57m: El.—08d 06h 31m: Sw.—08d 19h 53m: Wn Tl El.—08d 22h 35m: El.—10d 00h 30m: Wn.—11d 00h 06m: Wn Tl.—11d 03h 08m: Va.—12d 11h 15m: Va.—12d 12h 14m: El.—12d 12h 32m: El.—13d 04h 44m: El.—*16d 10h 18m:* thirty-three.—16d 12h 41m: El.—17d 11h 53m: So.—19d 07h 28m: Si.—19d 11h 09m: Ap.—19d 17h 10m: Tl.—20d 19h 26m: So.—21d 18h 33m: Wn.—23d 09h 22m: Ka.—23d 09h 41m: Fu.—23d 19h 47m: Do.—24d 01h 25m: Va.—24d 12h 12m: Fu Tl Va.—24d 19h 50m: So.—25d 15h 44m: El.—25d 16h 54m: El.—25d 21h 40m: Wn Tl El.—26d 18h 47m: So.—29d 20h 46m: Wa.—*30d 17h 47m:* twenty-eight.—30d 19h 43m: Tl El.

Preliminary report on solar-flare effects

Effects confirmed by ionospheric or solar observations are in italics.

1950 July 02d 17h 31m-46m: Eb.—*02d 19h 08m:* Hu.—09d 11h 35m: Hu.—09d

Geomagnetic planetary three-hour-range indices Kp, preliminary magnetic character-figures, and final selected days, July to September, 1950

E	July 1950 1	2	3	4	5	6	7	8	Sum
1	4o	2 –	2o	2o	3 +	2 +	2 –	3 +	20 +
2	2 –	3 –	2 –	1 –	1o	1 +	1 +	2 –	12o
3	0 +	1 –	1 –	1 +	3 –	3 +	4 +	5o	18 +
4	6o	5o	3 +	1 +	3 –	3 +	4 –	4o	29 +
5	4o	3 +	1 +	3 –	3 +	3 –	3 +	3 +	24o
6	3o	2 –	3 –	3 –	2 +	3o	1 +	2 –	18 +
7	3 –	2 +	1 +	2 +	2o	2 +	2o	4 –	19 –
8	2o	1 +	2o	2 –	3 –	2 –	0 +	0 +	12o
9	2o	4 –	4 –	2 –	3 +	3 –	2 +	1 +	21 –
10	1 +	1 +	2 –	2 +	2 +	3 +	2 –	2 +	16 +
11	4 –	2 –	1o	3o	5 +	4 –	5o	6o	29 +
12	7o	6 +	4 –	5 +	3 +	3 +	3o	3 +	35 +
13	5 –	4o	3o	5o	5 –	3 –	3o	2 –	29 –
14	2o	2 –	3 +	2 +	2o	2 +	3 –	3 +	20 –
15	3o	3 –	3 –	2 –	2 +	2 +	3 –	2 +	20 –
16	2o	3 –	2 +	4o	1 +	1o	1o	2 –	16o
17	3 –	1 –	3 –	2 –	1 +	1 +	2o	1 +	14 –
18	2o	1o	1 +	1 +	2o	2 –	1 –	1o	11o
19	2 –	1 –	0 +	1 –	2o	2o	1 –	1 +	9 +
20	2 –	2 +	1o	2 –	1 +	1o	2 +	2 –	13o
21	2 –	1 –	1 –	2 +	3o	2o	3o	3o	16 +
22	3o	3o	2 +	3 +	2o	1 +	2 –	1o	18 –
23	1o	1o	1o	0 +	0 +	0 +	0 +	0 +	5 –
24	3o	5o	4o	5 +	2o	3o	5o	5o	32 +
25	6o	6 +	5 +	4 –	4 +	3o	3 –	2o	33 +
26	1 –	1 –	1o	1o	2 –	1 +	2o	2o	10 +
27	1 –	1o	2 –	3 –	4o	2 –	2 –	2 –	15o
28	4o	3 +	1 +	1 –	1o	1 +	1o	3 –	15 +
29	3 –	2o	1 +	2 +	2o	2 +	3o	3 –	18 +
30	3 –	3 –	1 –	3 +	3 +	2o	3 –	2 –	19o
31	3 –	3 +	1 +	2 –	2o	3 +	3 +	3 +	21o

E	August 1950 1	2	3	4	5	6	7	8	Sum
1	4 –	3 –	3o	3 –	4 –	4 –	4 –	3o	26o
2	3 +	3o	3o	3 –	4 –	3 –	5o	4 –	27o
3	5 –	3 +	3 +	4o	4 –	4 –	3 –	3 –	28o
4	3 –	3 –	2o	2 +	2 +	2 +	2o	2o	18 +
5	1o	1 +	1o	1 –	2 –	2 –	2 +	3 –	12 +
6	4 –	2o	1 –	1 +	2o	2 +	3 +	3 +	19 –
7	3 +	3 +	2o	4 –	3 +	4o	5o	8 –	32 +
8	8 –	6 +	8 –	5o	3 +	3 –	3 –	4o	39 +
9	4o	5 –	5 –	4 +	5 –	5 –	4 +	5 +	37 –
10	4o	4 +	3o	3 +	5o	5 +	6 +	5o	36 +
11	4 +	4o	4o	5 –	4o	4o	4 –	4 +	33o
12	4 +	5o	5o	4 +	3 +	4o	3o	3 –	32 –
13	2o	3 +	3o	4 –	2 +	2 +	3 –	3 –	22o
14	3 –	3 +	3o	4o	5 +	3o	3 –	4o	28o
15	4 –	4 –	4 –	2o	2o	2o	2 +	1o	20 +
16	3 –	2o	2 –	1 –	1o	0 +	1 –	2 –	11 –
17	2o	1 –	0 +	0 +	1o	0 +	1 –	1o	6 +
18	2o	2o	1 +	1 +	2o	3 +	4o	5 –	21 –
19	6 –	4 +	6 –	6 +	8o	8 +	8 –	8o	54o
20	8 +	8o	8 –	8 –	4o	3 +	2 +	2o	43 +
21	3 –	1 +	5 +	4o	3o	3 +	3o	1 +	24o
22	2 –	1 +	2o	2o	2 –	2 +	2 +	1 –	14o
23	2o	1 –	1 +	3 +	2 –	2 –	1 +	3 –	15 –
24	1o	0o	1 –	2 +	1o	1 –	1 –	3 –	9o
25	3 –	2 –	1 –	1 +	1 +	1 –	0 +	1 –	9 +
26	0 +	0 +	1 –	1 –	0 +	0 +	0 +	1o	4o
27	2 +	1 –	1 –	1o	1 +	2 –	2 –	2 +	12 –
28	3 –	2 +	2o	4 –	3o	4 +	4 –	4 +	26o
29	3 +	5 –	3 +	3 –	4 +	3 +	3o	3 +	28o
30	4 +	4 –	3 +	1 +	2 –	3o	3 +	1 +	22o
31	1o	2 –	3o	2 –	2 +	3o	1o	1 +	15o

E	September 1950 1	2	3	4	5	6	7	8	Sum
1	1o	1 +	1 +	1 +	2 –	1 +	1 –	2o	11 –
2	2 –	2o	1o	0 +	1o	1 –	2 +	2 –	11 –
3	2o	4o	5 –	5 –	5 +	6 –	6o	7o	39 +
4	6o	4o	5 +	5 –	4 –	4o	5 –	5o	37 +
5	6o	5 +	6o	6 +	5 –	5o	5 +	5 +	44o
6	5 +	6 –	5 +	6 –	5o	4o	4 –	4o	39 –
7	3 +	3o	4o	4o	4o	3o	3o	3o	27 +
8	2 +	4 +	5 +	5 +	4 –	5 –	6 –	5o	36 +
9	5o	3 +	3 +	3 –	1 +	2 –	3o	3 –	23o
10	5o	3o	1o	2 –	3 +	4 +	4 +	3 +	26o
11	4 +	5 +	4 –	3o	1o	1 +	3o	4 –	25 +
12	3 +	1 +	3 +	3 +	3 +	1o	1 –	1 +	18 –
13	2o	2 +	2 –	2 –	1 +	3o	5 –	3 +	20o
14	1 +	1 +	1o	1o	1 –	1o	1 –	1 +	8 +
15	0 +	0o	1 –	1o	1o	1 +	0 +	1 –	5 +
16	2 +	1 +	2 –	4 –	4 –	5 +	4 –	3o	25 –
17	2o	3 +	4 –	3o	3o	4 +	4o	4 +	28 –
18	5 –	4o	5 –	4o	4o	3o	3 –	4o	31o
19	4 +	3 –	3 +	5o	5 –	2o	3o	5 +	30 +
20	6 –	5o	4o	5 –	4o	5 –	5 –	3 +	36o
21	4 –	4 –	2o	2 –	1 –	1 –	2 +	1 –	15 +
22	0 +	1 +	0 +	1 –	1o	2 –	2 –	1 –	8 –
23	1o	1 +	3 –	5 –	4o	4 +	5o	5o	28o
24	5 +	3o	2 –	3o	4 +	5o	5 +	6 +	34o
25	4 +	4o	5 –	4 –	4 –	6 –	4o	5 +	35 +
26	3 +	2 +	2 +	2o	2o	3o	5o	4 –	24 –
27	3o	3 –	3o	3 +	2 –	2 –	2 +	2 –	19 +
28	1 –	1 +	2 +	3 –	2o	1 +	2 –	2o	14o
29	1 –	0 +	1o	0 +	0 +	1 –	1o	1o	5 +
30	1o	1 +	1 +	2o	1 –	3o	4 +	4 –	17 +
31									

E	Preliminary C, 1950 July	Aug.	Sep.
1	0.6	0 8	0.0
2	0.2	1.0	0.4
3	0.9	0.9	1.6
4	1.3	0.4	1.5
5	0.7	0.4	1 6
6	0.5	0.6	1.3
7	0.5	1.6	0.9
8	0.3	1.8	1.5
9	0.7	1.3	0.8
10	0.5	1.4	1.0
11	1.3	1.1	0.8
12	1.4	1.0	0.4
13	1.1	0.6	0.7
14	0.6	1.0	0.0
15	0.6	0.6	0.0
16	0.5	0.1	1.0
17	0.2	0.0	1.0
18	0.2	0.9	1.0
19	0.3	2.0	1.2
20	0.3	1.9	1.3
21	0.6	0.8	0.4
22	0.6	0.4	0.0
23	0.0	0.4	1.2
24	1.4	0.2	1.4
25	1.4	0.1	1.3
26	0.2	0.0	0.8
27	0.6	0.2	0.4
28	0.6	0.8	0.3
29	0.6	0.9	0.0
30	0.6	0.7	0.8
31	0.6	0.5	

Final selected days July	Aug.	Sep.
Five quiet		
17	16	1
18	17	14
19	24	15
23	25	22
26	26	29
Five disturbed		
4	7	3
11	8	4
12	10	5
24	19	6
25	20	24
Ten quiet		
2	5	1
8	16	2
10	17	12
16	22	14
17	23	15
18	24	21
19	25	22
20	26	27
23	27	28
26	31	29

12h 34-45m: CF Eb Pi Do.—09d 18h 12-32m: Wn Eb SJ Ho Do.—11d 05h 38-50m: Wn.—12d 16h 07-17m: Tu.—12d 17h 06-20m: Eb.—*15d 04h 29m-05h 30m:* Ka.—*17d 13h 38-44m:* CF.—*18d 13h 12-24m:* CF.—19d 00h 33-40m: Ho ZS.—*19d 00h 33m-01h 10m:* Ka.—*19d 09h 36-41m:* CF.—19d 16h 08-53m: Eb.—21d 00h 46-52m: Ho.—*21d 13h 12-25m:* Wn.—*22d 15h 45m:* Hu.—24d 16h 39-54m: SJ.—27d 07h 31-36m: Eb.—*27d 09h 38m-10h 08m:* Wn.—27d 12h 35m-13h 30m: Ni Eb.—*29d 17h 15m:* Hu.

1950 August 01d 11h 20-50m: Tl.—01d 12h 15m: Va.—*02d 10h 48m:* Hu.—02d 22h 04-07m: Ho.—03d 01h 20-48m: ZS.—*03d 13h 30m-14h 10m:* CF.—04d 05h 50m-06h 00m: ZS.—04d 23h 30m-24h 30m: ZS Ho Ap.—06d 20h 02-25m: Eb Hu.—07d 01h 45m-02h 10m: ZS.—07d 03h 40m-04h 05m: Eb ZS.—*08d 01h 27-50m:* ZS.—*09d 00h 04-11m:* ZS.—09d 16h 11m: Va.—11d 17h 30m: Va.—14d 09h 14m: Va.—14d 12h 36m: Va.—*15d 17h 33m:* Hu.—*16d 01h 45m-02h 00m:* ZS.—16d 12h 18m: Va.—16d 23h 05-20m: Tu Ho ZS.—17d 02h 18-25m: ZS.—17d 13h 55m-14h 05m: Wn Pi.—18d 11h 40-54m: Ni Eb.—*19d 10h 06-15m:* CF.—*21d 01h 47m-02h 22m:* ZS.—23d 18h 06m: Pi.—25d 10h 06-30m: Wi CF Eb.—27d 12h 37-45m: Wn.—27d 23h 00-14m: ZS.—28d 13h 56m: Va.—31d 15h 24-37m: Eb.

1950 September 01d 16h 06-15m: Va.—06d 04h 41-45m: Eb.—07d 06h 41-46m: Eb.—07d 11h 20-23m: Eb.—07d 13h 43-48m: Eb.—09d 16h 16m: Hu.—10d 12h 02-08m: Eb.—*11d 11h 45m-12h 00m:* Hr.—12d 11h 32-45m: Va.—12d 12h 14-44m: Wi Eb.—13d 04h 42m-05h 15m: Tu.—14d 19h 54m-20h 04m: Tu.—15d 14h 05-20m: Wn, magn−, ion+.—19d 12h 08m: Hu.—19d 14h 32m: Hu.—*19d 17h 08-30m:* Wi CF Va.—*20d 03h 06-30m:* Ka.—20d 09h 35m: Ni.—21d 20h 32m: Hu.—26d 15h 20-45m: Hu Va.—30d 17h 47-56m: Ho.

COMMITTEE ON CHARACTERIZATION OF MAGNETIC DISTURBANCES

J. BARTELS, *Chairman*
University
Göttingen, Germany

J. VELDKAMP
Kon. Nederlandsch Meteorologisch Instituut
De Bilt, Holland

PROVISIONAL SUNSPOT-NUMBERS FOR OCTOBER TO DECEMBER, 1950

(Dependent on observations at Zurich Observatory and its stations at Locarno and Orosa)

Day	Oct.	Nov.	Dec.
1	41	78	82
2	41	62	80
3	41	57	77
4	50	67	75
5	50	79	61
6	45	94	46
7	54	80	85
8	78	55	108
9	84	61	94
10	79	60	94
11	68	46	100
12	88	48	115
13	75	42	94
14	72	61	79
15	106	81	59
16	103	42	42
17	99	66	26
18	74	58	19
19	50	50	7
20	48	36	0
21	27	22	7
22	20	18	0
23	22	16	13
24	32	20	31
25	30	26	39
26	37	32	56
27	51	64	58
28	55	74	35
29	95	69	41
30	107	73	23
31	74		43
Means........	61.2	54.6	54.5
No. days......	31	30	31

Mean for quarter: 56.8 (92 days)
Mean for year 1950: 83.3 (365 days)

M. Waldmeier

Swiss Federal Observatory
Zurich, Switzerland

CHELTENHAM THREE-HOUR-RANGE INDICES K FOR OCTOBER TO DECEMBER, 1950

[$K9 = 500\gamma$; scale-values of variometers in γ/mm: $D = 5.3$; $H = 2.6$; $Z = 4.2$]

Gr. day	October 1950 Values K	Sum	November 1950 Values K	Sum	December 1950 Values K	Sum
1	4565 5344	36	4456 5444	36	1331 1111	12
2	4655 4546	39	4344 2301	21	2422 2233	20
3	5553 3353	32	4333 1011	16	3212 1122	14
4	6544 4443	34	3565 3312	28	0211 0111	7
5	5454 3345	33	4333 1122	19	1122 2232	15
6	4254 1334	26	2122 0101	9	3244 2222	21
7	5453 3343	30	1010 0010	3	4321 1221	16
8	3322 2211	16	3211 1121	12	2122 2222	15
9	3234 2221	19	1320 0122	11	2322 1111	13
10	3111 1112	11	3345 4333	28	2322 1121	14
11	1111 0232	11	5342 3333	26	1311 1010	8
12	3521 1222	18	3353 3323	25	1432 2354	24
13	3331 2123	18	3443 3234	26	4354 2244	28
14	5434 4344	31	3221 1122	14	5421 2355	27
15	5432 2222	22	1011 1012	7	2222 2232	17
16	3355 4444	32	2110 1222	11	2323 3112	17
17	3343 3323	24	2232 2233	19	1224 1122	15
18	3344 2211	20	0012 3142	13	2122 1332	16
19	2211 0002	8	3311 1100	10	3232 1231	17
20	1023 1122	12	1111 2111	9	1113 1234	16
21	1010 1121	7	0000 0023	5	2111 2000	7
22	0101 1233	11	2120 3445	21	1212 4353	21
23	3323 3233	22	3011 1112	10	5455 3324	31
24	3233 1222	18	1011 2324	14	4453 2644	32
25	2100 0023	8	6535 6332	33	5444 3343	30
26	2222 1022	13	5566 4454	39	5445 3233	29
27	1011 1021	7	5655 5343	36	3443 3323	25
28	4556 6667	45	5544 4454	35	3242 2121	17
29	6655 5535	40	4423 3233	24	3223 1111	14
30	6544 4445	36	3333 1232	20	2112 2222	14
31	6444 4445	35			1221 1111	10

Ralph R. Bodle
Observer-in-Charge
Cheltenham Magnetic Observatory
Cheltenham, Maryland, U.S.A.

PRINCIPAL MAGNETIC STORMS

(Advance knowledge of the character of the records at some observatories as regards disturbances)

bservatory (Observer-n-Charge)	Green-wich date	Storm-time: GMT of begin.	Storm-time: GMT of ending[1]	Sudden commencement: Type[2]	Amplitudes[3]: D	Amplitudes[3]: H	Amplitudes[3]: Z	C-figure, degree of activity[4]	Maximal activity on K-scale 0 to 9: Gr. day	Gr. 3-hr. period	K-index	Ranges: D	Ranges: H	Ranges: Z
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
	1950	h m	d h		′	γ	γ					′	γ	γ
ege	Oct. 1	06 00	7 23	....	....	....	....	s	1	3	8	390	1780	1760
L. Cleven)	Oct. 28	02 40	2 17	....	....	....	....	s	28	4	8	370	2100	1420
									29	3, 4, 5, 6	8			
	Nov. 4	04 00	4 20	....	....	....	....	s	4	3	8	540	1990	2380
	Nov. 10	07 00	13 17	....	....	....	....	ms	10	3	7	250	1340	1400
	Nov. 24	17 00	30 12	....	....	....	....	s	25	4,5	8	470	2510	1280
	Dec. 12	05 26	14 04	s.c.*	−6	56	6	ms	13	3	6	150	930	820
	Dec. 22	10 40	28 15	....	....	....	....	ms	23	3	7	300	1800	1420
									24	4,5,6	7			
a	Oct. 14	02 ..	15 09	....	....	....	....	ms	14	5	6	54	406	462
H. Nelson)	Oct. 16	05 ..	18 12	....	....	....	....	s	16	4	8	113	1040	650
	Oct. 28	07 ..	2 17	....	....	....	....	s	28	4	8	160	1000	925
									31	5	8			
	Nov. 4	04 ..	4 20	....	....	....	....	s	4	3,4,5	8	262	850	900
	Nov. 10	03 ..	12 21	....	....	....	....	ms	10	3,4,5	7	115	890	520
	Nov. 24	17 ..	(*Note:* Not ended before next storm began.)					ms	25	4,5	7	91	810	890
	Nov. 26	08 32	29 18	s.c.	+30	185	+52	s	26	4	9	173	1270	810
	Dec. 6	06 ..	6 17	....	....	....	....	ms	6	4	7	51	590	370
	Dec. 12	05 29	14 05	s.c.	−3	20	+2	ms	13	3,5	6	53	350	405
	Dec. 14	15 ..	15 05	....	....	....	....	m	14	7,8	5	43	198	190
	Dec. 22	10 35	23 20	....	....	....	....	s	23	3	8	96	925	525
	Dec. 24	06 ..	28 12	....	....	....	....	ms	24	6	7	70	530	420
teveen	Sep. 30	17 46	8 18	s.c.*	−2	+38	−3	ms	3	7	7	50	365	125
van Sabben)									4	7	7			
	Oct. 14	02 40	18 12	....	....	....	....	ms	14	6	6	40	225	130
	Oct. 28	01 00	2 18	....	....	....	....	ms	28	7	7	65	315	215
	Nov. 4	02 00	5 24	....	....	...	....	ms	4	6	7	40	165	160
	Nov. 10	00 00	13 24	....	....	....	....	ms	10	3	6	40	210	70
	Nov. 22	12 00	23 03	....	....	....	....	ms	22	8	6	40	133	45
	Nov. 24	17 04	29 24	....	....	....	....	ms	25	5	6	45	250	130
									26	6,7,8	6			
									27	7	6			
									28	7	6			
	Dec. 12	05 26	14 24	s.c.	+2	+18	0	ms	13	7	7	42	220	81
	Dec. 22	11 00	27 24	....	....	....	....	ms	24	6	7	56	179	109
ltenham	Oct. 14	01 ..	16 04	....	....	....	....	m	15	1	5	22	65	40
R. Bodle)	Oct. 16	06 ..	18 12	....	....	....	....	m	15	3	5	17	76	32
	Oct. 28	02 ..	1 17	....	....	....	....	ms	28	7,8	7	31	107	145
									29	1	7			
	Nov. 3	01 ..	4 17	....	...	....	....	ms	4	3	6	34	118	46
	Nov. 10	02 ..	12 09	....	....	....	....	m	10	4	5	15	52	37
	Nov. 22	13 ..	23 01	....	....	....	....	m	8	5	5	8	76	29
	Nov. 24	23 ..	29 04	....	....	....	....	ms	26	4	6	34	67	57
	Dec. 12	05 27	14 24	s.c.	1	34	3	m	12	7	5	5	78	21
	Dec. 22	12 ..	28 00	....	....	....	....	ms	24	6	6	12	128	17

[1]Approximate time of ending of storm construed as the time of cessation of reasonably marked disturbance movements in the traces; more specifically, when the K-index measure diminished to 2 or less for a reasonable period.

[2]s.c. = Sudden commencement; s.c.* = small initial impulse followed by main impulse (the amplitude in this case is that of the *main impulse only*, neglecting the initial brief pulse); .... = gradual commencement.

[3]Signs of amplitudes of D and Z taken algebraically; D reckoned positive if towards the east and Z reckoned positive if vertically downwards.

[4]Storm described by three degrees of activity: *m* for moderate (when K-index as great as 5); *ms* for moderately severe (when K = 6 or 7); *s* for severe (when K = 8 or 9).

PRINCIPAL MAGNETIC STORMS—Continued

Observatory (Observer-in-Charge) (1)	Greenwich date (2)	Storm-time: GMT of begin. (3)	Storm-time: GMT of ending[1] (4)	Sudden commencement: Type[2] (5)	Sudden commencement: Amplitudes[3] D (6)	Amplitudes[3] H (7)	Amplitudes[3] Z (8)	C-figure, degree of activity[4] (9)	Maximal activity on K-scale 0 to 9: Gr. day (10)	Gr. 3-hr. period (11)	K-index (12)	Ranges: D (13)	Ranges: H (14)
	1950	h m	d h		′	γ	γ					′	γ
Tucson (J. B. Campbell)	Sep. 30	17 46	8 03	s.c.	0	+6	0	ms	1	3	6	24	135
									2	2	6		
									3	2	6		
									5	3	6		
	Oct. 13	19 ..	18 12	.....	....	....	....	ms	16	5	6	13	136
	Oct. 28	02 ..	2 17	.....	....	....	....	ms	28	4,5,6	6	23	158
									31	1	6		
	Nov. 10	01 ..	13 16	.....	....	....	....	m	10	5,6	5	14	120
									12	3	5		
	Nov. 24	23 ..	30 04	.....	....	....	....	ms	25	1,5	6	22	161
									27	2	6		
	Dec. 12	05 26	17 12	s.c.	0	+33	+1	m	12	7	5	12	130
									13	3	5		
	Dec. 22	11 ..	28 08	.....	....	....	....	ms	23	3	6	16	137
San Juan (P. G. Ledig)	Oct. 28	01 05	29 01	.....	....	....	....	ms	28	6	6	10	205
	Dec. 12	05 26	13 02	s.c.	0	+22	−4	m	12	7	5	7	160
	(*Note:* There was a very rough interval not considered to be a storm from Dec. 22, 11h, Dec. 26, 12h, with *K*-indices of 5 during the 7th and 6th periods of Dec. 22 and 24, respective												
Honolulu (R. F. White)	Oct. 1	00 00	5 13	.....	....	....	....	m	1	3	5	8	140
	Oct. 13	18 00	15 09	.....	....	....	....	m	14	5	4	6	117
	Oct. 27	19 00	1 18	.....	....	....	....	ms	28	4	6	9	159
	Nov. 24	22 00	28 23	.....	....	....	....	m	27	3,4	5	8	98
	Dec. 12	05 27	14 06	s.c.	0	25	12	m	12	3	4	5	132
	Dec. 22	11 00	26 13	.....	....	....	....	m	23	3	5	9	99
Zô-Sè (M. Burgaud)	July 24	01 50	25 18	s.c.	1	26	8	m	24	1	5	....	144
	Aug. 7	10 53	.. ..	s.c.	1	42	11	ms	8	2,3	6	9	186
	Aug. 14	8 20	.. ..	.....	−..	18	4	m	14	5	5	....	60
	Aug. 19	10 05	20 21	s.c.	....	20	4	ms	19	4	7	12	237
	Aug. 28	10 18	30 09	s.c.	....	20	3	m	29	2	5	....	....
	Sep. 3	8 18	6 15	.....	....	....	....	m	3	8	5	....	....
									4	3	5		
	Sep. 16	10 18	16 21	s.c.	1	34	9	m	16	5	5	....	100
	Sep. 17	11 55	.. ..	s.c.?	....	....	....	m	17	6	5	....	....
									18	5	5		
Instituto Geofísico de Huancayo (A. A Giesecke)	Sep. 30	17 46	6 00	s.c.*	−1	+83	+6	ms	1	5	6	8	314
	Oct. 14	00 30	15 03	.....	....	....	....	ms	14	5,6	6	9	365
	Oct. 16	05 50	17 22	.....	....	....	....	ms	16	5	6	11	295
	Oct. 28	01 04	1 19	.....	....	....	....	ms	28	5,6	6	12	577
	Nov. 10	01 18	12 21	.....	....	....	....	ms	10	6	6	7	207
	Nov. 24	17 11	30 04	.....	....	....	....	ms	26	5,6	6	12	250
	Dec. 12	05 27	14 23	s.c.	0	+49	+6	ms	12	7	6	(9)	347
	(*Note:* Ranges in parentheses doubtful; light burned out during recording hours.)												
	Dec. 22	10 45	27 05	.....	....	....	....	ms	22	6	7	8	282
Elisabethville (G. Heinrichs)	Oct. 28	01 10	29 01	s.c.*	....	....	....	ms	28	4,5,6,7,8	7	14	287
Apia (G. A. M. King)	Sep. 30	17 44	6 11	s.c.	....	+10	−2	m	1	3	5	7	175
									2	4,7	5		
									5	3	5		
	Oct. 14	02 39	15 09	.....	....	....	....	m	15	2	5	6	92
	Oct. 16	03 30	18 12	.....	....	....	....	m	16	5	5	5	72
	Oct. 28	01 09	2 15	.....	....	....	....	ms	28	4	6	7	160
	Nov. 4	04 40	5 06	.....	....	....	....	m	4	3,4	5	5	64
	Nov. 10	01 15	10 15	.....	....	....	....	m	10	3	5	5	129
	Nov. 22	12 07	23 06	s.c.	−..	−5	−2	m	22	8	5	6	54
	Nov. 24	21 30	29 15	.....	....	....	....	m	26	4	5	9	106
									28	7	5		
	Dec. 12	05 27	14 05	s.c.	+1	+26	−9	...	12	2	5	....	....
			(*Note:* No record on Dec. 13 in *D* and *H*.)										
	Dec. 22	10 35	23 11	s.c.	−..	−15	+5	m	22	5	5	11	102
									23	1	5		
	(*Notes:* No record on Dec. 22 in *Z*. Time of s.c. 00h 17m on Dec. 23.)												

PRINCIPAL MAGNETIC STORMS—Concluded

Observatory (Observer-in-Charge) (1)	Greenwich date (2)	Storm-time: GMT of begin. (3)	Storm-time: GMT of ending[1] (4)	Sudden commencement: Type[2] (5)	Amplitudes[3] D (6)	Amplitudes[3] H (7)	Amplitudes[3] Z (8)	C-figure, degree of activity[4] (9)	Maximal activity on K-scale 0 to 9: Gr. day (10)	Gr. 3-hr. period (11)	K-index (12)	Ranges D (13)	Ranges H (14)	Ranges Z (15)
	1950	h m	d h		′	γ	γ					′	γ	γ
manus	Sep. 30	17 47	8 16	(*Note:* Temporary failure of electric supply.)				m	1	5	5	25	134	98
M.		(Ap-							2	7	5			
Wijk)		prox.)							3	7	5			
									4	7	5			
									7	5	5			
	Oct. 13	19 40	15 16	.....	....	....	....	ms	14	6	6	20	135	115
	Oct. 16	09 ..	17 22	.....	....	....	....	m	16	4	5	17	107	67
	Oct. 28	01 05	2 00	.....	....	....	....	ms	28	7	6	41	208	157
	Nov. 4	04 ..	4 20	.....	....	....	....	m	4	6	5	18	154	103
	Nov. 22	12 06	23 00	s.c.	−1	−8	−9	m	22	6	5	13	55	85
	Nov. 24	17 04	30 00	s.c.	−1	−2	−3	ms	26	4	6	25	140	101
	Dec. 12	05 26	16 15	s.c.	−1	+7	+6	m	12	3,7	5	19	175	103
									13	7,8	5			
									14	7,8	5			
	Dec. 22	09 ..	28 18	.....	....	....	....	ms	22	7	6	35	130	145
berley	Sep. 30	17 48	6 12	s.c.*	+1	+9	−4	ms	2	4	6	31	156	100
W. Beagley)	Oct. 14	02 38	15 09	s.c.*	+1	+26	−4	m	14	5	5	23	131	55
	Oct. 16	00 00	18 12	.....	....	....	....	m	16	4,5	5	20	115	71
	Oct. 28	01 13	2 16	.....	....	....	....	ms	28	3,4,5	6	33	194	118
	Nov. 4	00 41	5 03	.....	....	....	....	ms	4	3	6	20	155	74
	Nov. 10	01 23	11 09	.....	....	....	....	m	10	3,4,5	5	22	123	78
	Nov. 11	20 48	13 16	.....	....	....	....	m	12	3	5	16	111	42
	Nov. 22	12 05	23 01	.....	....	....	....	m	22	6,8	5	19	171	40
	Nov. 24	16 49	30 08	.....	....	....	....	ms	26	4	6	24	200	77
	Dec. 5	05 45	7 08	.....	....	....	....	m	6	4	5	14	95	33
	Dec. 12	05 25	14 05	s.c.*	+2	+66	−11	m	12	2	5	18	180	50
	Dec. 14	05 41	16 13	s.c.*	+1	+18	−3	m	14	2,7	5	15	145	29
	Dec 22	10 54	27 12	.....	....	....	....	m	23	3	5	21	160	72
									26	3	5			

CORRECTION TO THE PAPER "ON THE EFFECT OF A CURRENT OF IONIZED AIR UPON THE EARTH'S MAGNETIC FIELD"

Dr. M. H. Johnson has called attention to a serious error in my paper which appeared in the September 1950 issue of this JOURNAL (**55**, No. 3, pp. 247-260). The analysis of the second half of the paper, §§VI to X, rests upon consideration of the first term of a series development of the general solution (4.3). I neglected to observe that while the problem of calculating the steady perturbation field is equivalent to that of calculating three Newtonian potentials, as indeed I stated, nevertheless the total masses of the respective mass distributions are zero. Thus what I called the *strength* of the disturbance is actually zero, as can be seen by combining (6.3) with (4.2) and (3.7)

$$\mathfrak{M} = \int_{\mathfrak{v}^*} \mathbf{M}\, d\mathfrak{v} = \frac{\mu\sigma}{c^2} \int_{\mathfrak{v}^*} \mathbf{curl}\, (\mathbf{v} \times \mathbf{H}_0)\, d\mathfrak{v}$$

$$= \frac{\mu\sigma}{c^2} \oint_S d\mathbf{S} \times (\mathbf{v} \times \mathbf{H}_0) = 0$$

since the disturbance is assumed to vanish outside a finite region. The first term in (6.2), upon which I based the remainder of the analysis, is thus zero. This fact invalidates my specific conclusion (5) and the second half of the specific conclusion (4) in §IX, besides reducing many of the equations to the form $0 = 0$. In the examples given in §§VII and VIII, the effective surface-charge associated with the discontinuity on the boundary surface will yield a strength equal and opposite to that there calculated, so that again the total strength is zero.

The graphical procedure for finding the strength of disturbance outlined in §VI may not be useless, nevertheless. Should this procedure when applied to observatory data yield a non-zero value for any component $\mathfrak{M}_e$ of $\mathfrak{M}$, this result would imply that transients and electromagnetic waves could not be neglected in analyzing the disturbance in question.

I hope in a later paper to be able to calculate explicitly the first non-vanishing term in the series expansion (6.2) and thus to obtain useful formulae of the type suggested in §VI.

C. TRUESDELL

NAVAL RESEARCH LABORATORY
Washington 20, D.C., January 26, 1951

NOTES

(1) *Spring meetings, International Scientific Radio Union*—The regular joint spring meeting of the U.S.A. National Committee of the International Scientific Radio Union (URSI) and the Professional Group on Antennas and Wave Propagation of the Institute of Radio Engineers will be held on April 16, 17, and 18, 1951, at the National Bureau of Standards, Washington, D.C. Sessions will be held on ionospheric radio propagation, terrestrial radio noise, and radio waves and circuits.

(2) *Expansion of geophysical activities in Australia*—It will be of interest to the readers of the JOURNAL to know that the Australian Government is building a substantial geophysical organization within the Bureau of Mineral Resources, Geology and Geophysics. Dr. J. M. Rayner, Chief Geophysicist of that Bureau, revealed that it is the aim to have about 100 geophysicists working in both pure and applied fields, together with draftsmen, computers, instrument makers, field assistants, etc. New equipment is also being obtained, including modern gravity meters, seismographs, and airborne magnetometers. Magnetic and gravity surveys of Australia, New Guinea, some Pacific and sub-antarctic islands, and, ultimately, Antarctica, are contemplated. It is the intention to continue the operation of the magnetic observatories at Watheroo, Toolangi, Macquarie Island, and Heard Island, and to construct a major geophysical observatory at Port Moresby, Papua. Other activities will include geophysical investigations for coal, oil, natural gas, and underground water; investigations of the resources of uranium and thorium; engineering and military geophysics; investigations on the structure of the Great Barrier Reef and crustal layering in New Guinea; geophysical investigations in the sub-antarctic and antarctic areas; and development and research in the Geophysical Laboratory which has been established in Melbourne.

(3) *Alaskan Science Conference held*—A conference of United States and Canadian scientists was held in Washington, D.C., November 9 to 11, 1950, under the auspices of the National Academy of Science-National Research Council. The purposes of the Conference were to stimulate wider interest in Alaskan research and to explore ways and means by which those who are actively engaged in field research in Alaska could coordinate and publicize their investigations. One division of the Conference was devoted to that of the physical sciences—geology and geography, geophysics, meteorology, and oceanography. Six papers were presented before the section of geophysics on November 9, 1950.

(4) *Meeting of Society of Exploration Geophysicists*—A regional exploration meeting was held in Fort Worth, Texas, November 30 and December 1, 1950, jointly sponsored by the Fort Worth Geophysical Society, Fort Worth Geological Society, Dallas Geophysical Society, Geophysical Society of Tulsa, the Ark-La-Tex Geophysical Society, and the Permian Basin Geophysical Society. The objective of the meetings was the discussion of exploration problems, both geological and geophysical, of a number of oil provinces in the south-central United States. Over a thousand exploration scientists were in attendance to hear the 18 technical papers on the program.

(5) *New institute*—The Ohio State University Research Foundation has established the Institute of Geodesy, Photogrammetry and Cartography. Dr. W. A. Heiskanen has been appointed Scientific Director of this new Institute. Prof. Earl Church and Mr. Walter D. Lambert will assist him in the instruction and research to begin next fall.

(6) *New Japanese journal*—A new journal, a collection of results of research by the members of the Meteorological Research Institute, Tokyo, will be published quarterly in the future under the name "Papers in Meteorology and Geophysics." Enlarged in April 1947, the Institute now is composed of seven laboratories, among which is the Geoelectric and Geomagnetic Laboratory. The publishers (Meteorological Research Institute, Mabashi, Suginami, Tokyo, Japan) will gladly exchange with other periodicals in this field of science. Volume 1, No. 1 (Oct. 1950) has been issued and contains 164 pages.

(7) *Magnetic disturbance, aurora borealis, and solar halo*—The United States *Hydrographic Bulletin* (Nos. 3193, 3196, and 3197, of November 18, and December 9 and 16, 1950) contained the following accounts of a magnetic disturbance in the Caribbean Sea, and aurora borealis and solar halo in the North Atlantic:

Captain N. Pontikos, Master of the American S.S. *Gulf Merchant*, reported that on October 7, 1950, from latitude 24° 00′ north, longitude 86° 10′ west to latitude 22° 00′ north, longitude 85° 06′ west, both the magnetic and steering compasses fluctuated radically, as much as 10° to 12°. The weather was partly cloudy with a good deal of lightning in the SE; wind S, force 4; barometer 29.93 inches; air temperature, 83°F.

Second Officer L. T. Oletti of the Italian S.S. *Eugenio C.*, Captain S. Luchetta, Master, reported that on August 20, 1950, at $01^h\ 30^m$ GMT, in latitude 34° 10′ north, longitude 60° 30′ west, about one hour after moonset, a considerable amount of light was observed above the northern horizon. From time to time the luminosity increased and colored shafts rose from the horizon to an altitude of 40°. The color was light brown toward the west, varying to a blue-white on the eastward side. About an hour before sunrise, the phenomenon gradually disappeared. The weather was clear; wind NNW, force 3; barometer 30.2 inches; air temperature 79°F.

Mr. G. Arnoux, an officer aboard the French S.S. *Ile de France*, Captain J. Leveque, Master, reported that on October 28, 1950, at $23^h\ 30^m$ GMT, in latitude 46° 35′ north, longitude 52° 00′ west, a brilliant display of aurora borealis was observed. Three concentric ellipses with short beams of white light, accented occasionally by yellow, green, and purple irridescence, illuminated the northern sky from 270° to 050°, reaching from an altitude of 5° to 15°. At $00^h\ 10^m$ some parts of the outer ellipse developed into beams ascending toward the zenith. At $00^h\ 15^m$ the phenomenon faded, until only a few patches of light remained. The sky was clear and visibility very good; wind NW, force 4; barometer 30.41 inches.

Third Officer J. H. Greenberg of the American S.S. *American Scout*, Captain A. Horka, Master, reported that on September 11, 1950, at $17^h\ 35^m$ GMT, in latitude 43° 24′ north, longitude 52° 06′ west, an unusual multicolored sun halo was observed with colors as vivid as those in a rainbow. The halo had a radius of 22° 10′.5 and first appeared when the sun was at an altitude of 42° 12′.5. The phenomenon remained visible until $17^h\ 50^m$. The sky was partly cloudy with alto-cumulus, cirro-

cumulus, and cirro-stratus formations; visibility good; wind NNW, force 2; barometer 30.16 inches, steady; air and sea temperature 62°F.

(8) *Five Coeur d'Alene, Idaho, aeromagnetic maps released*—Aeromagnetic Geological Survey maps of the Coeur d'Alene mining district are available for publication inspection. This group of maps, scaled at approximately $2\frac{1}{2}$ inches to the mile (1 to 24,000) shows magnetic contours compiled from a total intensity aeromagnetic survey made in 1948. The magnetic contours have been superimposed on the Geological Survey topographic maps to provide ground control. This work was undertaken in order to determine the magnetic features of an important mining district and to study their relation to geologic features, especially in respect to the occurrence of lead and zinc ore with associated pyrrhotite.

(9) *Aeromagnetic maps of Rowe-Mora Basin, New Mexico*—Four aeromagnetic maps of a 2,500 square-mile portion of the Rowe-Mora Basin in New Mexico have been published by the Geological Survey. These maps, on a scale of one inch equals one mile, were compiled from an aeromagnetic survey made in 1946. Variations of total magnetic intensity from an arbitrary base level are shown by magnetic contours. The flight level of the aircraft was approximately 1,000 feet above the ground. The magnetic survey was made to obtain information on the configuration, composition, and approximate depth of the crystalline rocks underlying the sedimentary rocks in the area. Such information may aid in deciphering the structure of the sedimentary rocks and consequently in exploration for petroleum.

(10) *Aeromagnetic maps of eleven Indiana counties released*—Aeromagnetic maps of Boone, Clinton, Gibson, Hendricks, Montgomery, Perry, Putnam, Tippecanoe, Vanderburgh, Vermillion, and Warren counties, Indiana, have been published by the Geological Survey as geophysical investigations, maps GP35 to GP45, respectively. These maps, covering 4,500 square miles in west-central and southwestern Indiana, were prepared by W. J. Dempsey and J. R. Henderson in cooperation with the Indiana Department of Conservation, Division of Geology. They are part of a series that will cover the entire State. Each map—scale 1 inch equals 1 mile—shows by contours the variation in total intensity of the earth's magnetic field, as measured by the airborne magnetometer flown 1,000 feet above the ground. This information is expected to aid in the search for geologic structures favorable to the accumulation of petroleum. Three-fourths of Indiana has now been aeromagnetically mapped, and the results are available as preliminary maps of 26 counties in open file, and as published maps of 46 additional counties.

(11) *Commemoration, Journal of the Franklin Institute*—The Journal of the Franklin Institute, one of the oldest scientific publications in the United States, is to be congratulated on its 125th anniversary. Its Janaury, 1951, issue contains a brief history of the Journal and an interesting group of papers ("Science and Tomorrow") predicting future developments in many fields of science and technology.

(12) *Geomagnetic activities of the United States Coast and Geodetic Survey*—Mr. S. R. Zegers, of the Inter-American Geodetic Survey, received training in magnetic field observations at the Cheltenham Observatory prior to beginning work in South America. Mr. J. A. Kozlosky, of the Inter-American Geodetic Survey, has restandardized his field equipment at the Cheltenham Observatory after about one

year in South America. The equipment will be completely overhauled prior to the start of extended field work in Brazil.

Dr. Takesi Nagata, of the Geophysical Institute of Tokyo, and Dr. Kiyoo Wadati, Director of the Central Meteorological Observatory in Tokyo, visited the United States Coast and Geodetic Survey office.

Sr. J. J. Recasens, of Cuba, has been a guest of the United States Coast and Geodetic Survey, studying the methods and instruments used in magnetic work.

A photoelectric, remote-recording, three-component magnetograph has been in operation at Cheltenham Observatory for two months. Results have been extremely satisfactory, with practically zero drift for the two-month period.

Captain E. B. Roberts, Chief, Division of Geophysics, United States Coast and Geodetic Survey, reports, after serving as United States delegate at the Fifth General Assembly of the Pan American Institute of Geography and History, held in Santiago, Chile, October 1950, that excellent interest and activity was manifested by several American Republics at the meetings of the subcommittees on Gravimetry and Geomagnetism and on Seismology. Definite action was taken in several respects to promote and coordinate geophysical work in the American Republics. The great activity of Argentina was particularly evident.

(13) *Handbook, Physical Society, London*—A handbook of the Physical Society's 35th exhibition of scientific instruments and apparatus, 1951, was published at the beginning of March 1951. Price of the handbook by post is 6 shillings, and orders with remittances should be sent to The Physical Society, 1 Lowther Gardens, Prince Consort Road, London, S.W. 7, England.

(14) *Current Kp diagrams*—Current *Kp* diagrams, in the usual note-script, arranged in solar rotations for the detection of recurrence tendencies, will be issued monthly by the Geophysikalisches Institut, Herzberger Landstrasse 180, (20B) Göttingen, Germany, with the consent of the Committee on Characterization of Magnetic Disturbances of the IATME.

(15) *Personalia*—Dr. *A. M. van Wijk* reported that the annual publication for Hermanus Magnetic Observatory, South Africa, for the years 1947-48 is now in the printer's hands; also that the special piers to eliminate various effects on the seismic instruments were completed, and that two Milne-Shaw seismometers have been installed.

Dr. *Takesi Nagata*, of the Geophysical Institute, Tokyo University, Tokyo, Japan, arrived in Washington, D.C., October 20, 1950. He was a guest investigator at the Department of Terrestrial Magnetism, Carnegie Institution of Washington, for a three-month period. Dr. Nagata left Washington on January 12, 1951. Before returning to Japan, he will visit the Woods Hole Oceanographic Institute, the University of Chicago, and the California Institute of Technology.

Dr. *M. Hasegawa*, professor at Kyoto University, Japan, was awarded the Academy prize of the Imperial Academy of Japan in March 1950, for his investigations of the diurnal variation of the terrestrial magnetic field.

Mr. *Donald B. Harris* has been named technical assistant to the president of Airborne Instruments Laboratory, Mineola, New York. He was formerly executive assistant to the director of research of Collins Radio Company, Cedar Rapids, Iowa.

Dr. *Kiyoo Wadati*, of the Central Meteorological Observatory, Tokyo, Japan,

and Prof. *Satio Hayakawa*, of the University of City Osaka, Japan, visited the Department of Terrestrial Magnetism, Carnegie Institution of Washington, on September 8, 1950.

Mr. *Yuichiro Aono*, Chief of the Ionospheric Propagation Section, Central Radio Propagation Observatory, and member of the Radio Regulatory Commission, Tokyo, Japan, spent some time in Washington, D.C., around December 1, 1950, visiting the Bureau of Standards, Department of Terrestrial Magnetism, and other scientific establishments.

Dr. *J. C. R. Heydenrych*, of the National Physical Laboratory, South Africa, visited the Department of Terrestrial Magnetism, Carnegie Institution of Washington, on January 19, 1951.

Prof. *Sydney Chapman*, on leave from Oxford University, visited the Geophysical Institute, University of Alaska, during March 8-11, 1951, where he delivered lectures on "Northern lights" and "Magnetic storms."

We are sorry to record the death, at Göttingen, Germany, on 2 February 1951, of Prof. *Hans Gerdien*, in his 74th year. He was well known for his work in atmospheric electricity. He designed the equipment now almost universally used in measuring the conductivity of the atmosphere.

Dr. *Leland B. Snoddy*, of the Rouss Physical Laboratory, University of Virginia, died on November 12, 1950, at the age of 52. He was noted for his researches dealing with transient phenomena in vacuum arc and spark discharge, spark discharge in air at atmospheric pressure, photographic studies of lightning, separation of isotopes by ultracentrifugal force, and acceleration of ions and electrons.

Prof. *Hantaro Nagaoka* died on December 11, 1950, at the age of 84. He was a former President of Osaka University and of the Imperial Academy of Japan, and an honorary professor of both Tokyo and Osaka universities. His fields of investigation were many, including gravity, seismology, the ionosphere, magnetism (atomic theory), electricity (hydrodynamics), and optics.

LIST OF RECENT PUBLICATIONS

By W. E. Scott

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